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Sunlight and Skylight as Determinants of Photographic Exposure.[a] I. Luminous Density as Determined by Solar Altitude and Atmospheric Conditions

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The various factors involved in the determination of correct camera exposure are discussed in detail. By making certain assumptions concerning the average characteristics of cameras and the average amount of flare light attributable to a scene having normal or average luminance distribution characteristics, a simplified exposure formula is deduced. This shows the relation between correct *camera exposure*, the *minimum scene luminance*, and the *speed* of the photographic material.

During the past few decades a large volume of reliable data relative to the amount of light reaching the earth's surface from the sun and sky has been accumulated by various observers. There is little evidence that this information has been used to any extent in the compilation of extant exposure guides, exposure tables, exposure computers, etc. This wealth of information which is in the literature has been examined critically. By using all of this information and applying appropriate weighting factors, an average has been obtained which, it is believed, represents with sufficient precision for photographic purposes the amount of light available under various atmospheric conditions and for various hours of the day, months of the year, and geographical latitudes.

The problem of how the amount of light reaching the earth's surface from the sun and sky can be most significantly evaluated for photographic purposes is discussed at some length. It has been shown quite conclusively in previous publications that the minimum scene luminance is the most reliable criterion of correct camera exposure when a negative-positive photographic process is being used. It follows, therefore, that the most satisfactory method of evaluating the amount of sunlight and skylight reaching the earth's surface is one which correlates most precisely with minimum scene luminance. There seems to be no justification for assuming the predominance of any particular orientation of scene elements. Hence, it seems improbable that the evaluation of the solar light in terms of any particular plane can be expected to give the desired correlation. Computation of the relationship between solar altitude and the illuminance incident upon the *horizontal*, the *perpendicular*, and the *normal* planes indicates that these values do not correlate satisfactorily with practical experience or with the theoretical conclusions. A relatively new concept, the *volume density of luminous energy*, is therefore introduced. This is an evaluation of the amount of luminous energy reaching the earth's surface from the sun and the sky irrespective of the direction from which

[a] Relatively soon after the entrance of the United States into World War II, the American Standards Association issued as an American emergency standard a "Photographic Exposure Computer" (Z38.2.2–1942), prepared at the request of the U. S. Army and Navy. While this computer was intended primarily for use by the armed forces, it was also made available to civilians.

This computer consists of tables of the light values for various geographical latitudes, months of the year, and hours of the day, and of tables giving indices for different types of scenes and sky conditions. By selecting the correct numbers from these tables and having the proper exposure index for the negative material being used, the correct exposing conditions can be determined. To simplify this computation, a small circular slide-rule type of calculator is supplied as a part of the computer.

The data relative to light index, scene index, etc., in this standard are presented in extremely condensed form and little information is given on the source of this material. Some users of the computer may be interested in knowing upon what foundation of physical and psychophysical data the computer is based. Since the compilation of the data for this computer was carried out largely in these Laboratories, the American Standards Association has requested that a treatise be published setting forth the basis for the data used in the body of the computer.

it arises. It is concluded that this method of evaluation gives better correlation with minimum scene luminance than any of the other methods which have been tested. The validity of this conclusion is supported by both theoretical and practical considerations. Using the values of luminous density, tables of *light indices* have been prepared covering various hours of the day, months of the year, and geographical latitudes.

The modification of the luminous density values applying to the clear atmospheric condition by atmospheres which are not clear is discussed. While there are several causes of diminution in luminous density, by far the most important one is the presence of condensed water vapor in the atmosphere which gives rise to the conditions commonly referred to as haze, cloud, etc. The available data on the quantitative relationship between luminous density at the earth's surface and the amount of haze and clouds present in the atmosphere is discussed and summarized. Some of these conditions can be described with sufficient precision so that they can be recognized with fair certainty. Conditions describable as *light haze*, *medium haze*, *heavy haze*, *light cloud*, *medium cloud*, and *heavy cloud*, fall in this category. For some of these conditions, the experimental data are sufficient to permit the computation of *atmospheric indices* of considerable reliability. In other cases by interpolation, extrapolation, and estimate, values of atmospheric index which are believed to be sufficiently precise for photographic purposes have been obtained.

OUTLINE

I. INTRODUCTION

IN the practice of photography one of the problems which the operator continually faces is that of exposure. Whenever a picture is to be made he must decide what exposure should be given so that a negative with the desired characteristics will be obtained. This problem can be solved rigorously by precise scientific methods. This requires a means of measuring the luminance characteristics of the scene, information regarding the speed of the negative material being used, and a knowledge of the optical and mechanical characteristics of the lens-camera system being employed. Of the data required for a precise solution of the exposure problem, those relating to the luminance of the scene are the most difficult to obtain. In practical field work it is frequently impossible, or at least not feasible, to make measurements of luminance. Fortunately, modern negative materials have sufficient latitude so that considerable deviation from the correct values can be tolerated without appreciable loss of negative quality. In fact, many expert operators who are familiar with the characteristics of negative materials and of the lens-camera equipment have learned by long experience to estimate the luminance characteristics of the scene visually with sufficient precision to obtain negatives of a uniformly high quality. Less expert operators and amateurs (and even experts when working under unusual and unfamiliar lighting conditions) usually need some assistance in obtaining a sufficiently precise value of scene luminance. For this purpose various forms of "exposure meters" have come into extensive use during recent years. However, because of the cost or unavailability of such instruments there is and has been for many years a demand for so-called "exposure tables" which assist the operator in obtaining a sufficiently precise value of scene luminance for a particular set of conditions. Such exposure tables usually give numbers representing the amount of light available at various hours of the day and months of the year at different geographical latitudes. These numbers, when combined with others based upon atmospheric conditions and scene structure, yield a value indicative of the scene luminance. Moreover, to facilitate the computation involving the various numbers which must be combined to give the final value representing scene luminance, the different factors are frequently arranged in tabular form or a simple computing device of the slide-rule type is provided. The final result obtained by these devices is embodied in two numbers: the exposure time, t, and the diaphragm setting, f (aperture ratio of the camera lens), which should produce a negative of good quality, at least insofar as exposure is concerned.

Exposure tables are not new. Many forms have been in use for several decades and some of them have apparently given good results, judged by the testimony of their users and by the quality of negatives obtained. However, when an attempt is made to appraise their validity in terms of the quality and generality of the fundamental physical data on which they are based, many difficulties are encountered. In many cases no published information can be found which explains (or gives substantiating evidence) why some particular diurnal, annual, or geographical latitude variation in scene luminance is assumed. In some cases where measurements of luminance are quoted as a basis for the assumed variation, it is found that they go back three or four decades to a time when techniques for precise measurements of light intensities were unknown or imperfectly developed. A recent publication by Berg[1, b] throws an interesting light upon the methods by which some of the existent exposure tables have been evolved.

When the authors of this communication became interested in this subject, a search of the literature revealed little evidence that the vast amount of highly reliable modern data (accumulated during the last thirty-odd years) relative to the amount of radiant energy reaching the earth's surface from the sun had been utilized or even recognized by compilers of exposure tables. It seemed profitable, therefore, to devote some further study to the subject.

Admittedly there are several approaches to the problem of photographic exposure and to the evaluation (in terms of hour of day, month, and geographical latitude) of the amount of light available for photographic purposes. Even so, there is no justification for ignoring the more recent and undoubtedly the most precise measurements of solar radiant energy.

In this country we are indebted for these data largely to Abbot and Fowle and their co-workers at the Smithsonian Institution and to Kimball and his associates of the U. S. Weather Bureau. Some further data, to which reference will be made later, are available from other sources in this country and abroad. The two sources just mentioned provide a large portion of all the recent reliable high precision measurements now available on the intensity of solar radiant energy at the earth's surface. These results have been published from time to time during the last two or three decades and reference to particularly relevant ones will be given later.

An excellent summary of the data relating to this subject has been made recently by Moon[2, 3] who, after carefully averaging and weighting all the available evidence, has proposed certain solar radiation curves for adoption as standards for engineering use. While much of Moon's interest in this subject is devoted to the establishment of a standard spectroradiometric curve for direct solar radiant energy, he has performed a very useful service by converting these radiometric data to visual units, thus giving material which can be applied directly to the problem of photographic exposure.

The possibility of using either the radiometric or the photometric approach in dealing with photographic exposure deserves some comment. It is true that photographic materials in general have a sensitivity to radiant energies of different wave-lengths which departs appreciably from the

[b] Numbered references will be found at the end of the article.

spectral sensitivity of the human eye. It might be argued, therefore, that radiometric units should be used in treating the photographic exposure problem. Such a course, however, is not justifiable when it is realized that the majority of pictures are made with the purpose of reproducing, either truthfully or with some desired modification, the *visual* aspects of the scene. Visual characteristics of a scene are specifiable in terms of its luminance distribution. The faithfulness, or the lack of it, with which the final photographic positive reproduces these characteristics is also expressible only in terms of the luminance of the positive. It seems most rational, therefore, to express the sensitivity characteristics of photographic materials in terms of photometric rather than radiometric units. It must be admitted that in some cases the photographic materials are used to record radiant energies lying outside the visible spectrum and under such conditions it is necessary to resort to the use of radiometric methods and units. This discussion, however, is concerned entirely with the use of photographic materials for the reproduction of a visual appearance and hence photometric terms and units are most appropriate.

TABLE I.* Radiometric and photometric terminology.

	Name	Symbol	c.g.s.	m.k.s.
	Radiometric (Physical)			
*R*1	Radiator (source of radiant energy)			
*R*2	Radiation (process)			
*R*4	Radiant energy	*U*	erg	joule
*R*5	Radiant density	*u*	erg/cm³	joule/m³
*R*6	Radiant flux	*P*	erg/sec	watt
*R*7	Radiant emittance	*W*	erg/sec/cm²	watt/m²
*R*8	Radiant intensity	*J*	erg/sec/ω	watt/ω
*R*9	Radiance	*N*	erg/sec/ω/cm²	watt/ω/m²
*R*10	Irradiance	*H*	erg/sec/cm²	watt/m²
	Photometric (Psychophysical)			
*L*1	Luminator (source of luminous energy)			
*L*2	Lumination (process)			
*L*4	Luminous energy	*Q*	lumerg	talbot
*L*5	Luminous density	*q*	lumerg/cm³	talbot/m³
*L*6	Luminous flux	*F*	lumerg/sec	lumen
*L*7	Luminous emittance	*L*	lumerg/sec/cm²	lumen/m²
*L*8	Luminous intensity	*I*	lumerg/sec/ω	lumen/ω (candle)
*L*9	Luminance	*B*	lumerg/sec/ω/cm²	lumen/ω/m² (candle/m²)
*L*10	Illuminance	*E*	lumerg/sec/cm²	lumen/m² (lux)

K = Ratio of photometric to the corresponding radiometric term, thus $K = L6/R6$ = Luminous efficiency (lumens/watt) of the radiant energy involved

* The nomenclature given in this table and used in this communication differs in many details from the nomenclature recommended by the Illuminating Engineering Society and approved by the American Standards Association. These modifications of the standard nomenclature were proposed by the Colorimetry Committee of the Optical Society of America (in 1937 and 1944) as somewhat simpler and more systematic, with hope that they might be considered in any revisions of the standard nomenclature. The letter symbols shown in the table are identical with those adopted by the American Standards Association for the corresponding concepts. The symbol ω denotes a unit solid angle, the solid angle subtended by one square meter of the surface of a sphere having the radius of one meter.

While it is true that the amount of solar radiant energy available at the earth's surface for photographic purposes is most usefully expressed in terms of its visual characteristics (i.e., photometric units), we will find it necessary at times to use the corresponding radiometric units. This necessity arises because a large proportion of the great volume of measurements of sunlight and skylight, which have been made by meteorologists, astrophysicists, illuminating engineers, and others during the past three or four decades, are in terms of radiant energy rather than of luminous energy. In the past a considerable diversity of terminology has been employed in the radiometric and photometric fields with the result that much confusion has existed. However, in 1937 the Colorimetry Committee of the Optical Society of America published a preliminary report[4] in which a simplified, systematic, and correlated system of terms covering both radiometric (physical) and photometric (psychophysical) concepts was proposed. More recently in the final draft of one of the chapters of the report of this committee, The Psychophysics of Color,[5] the same terminology (with a few minor modifications) together with letter symbols and defining formulas was again published with recommendations that it be adopted for general use. In the present communication we will adhere as closely as possible to the terminology recommended by the Colorimetry Committee of the Optical Society of America.

For the convenience of those readers who may find access to the two previous literature references somewhat difficult, this terminology is shown in Table I. The form in which the defining formulas are given has been modified slightly from that shown in the original publication. There are some differences of opinion among experts in this field as to the most suitable form for expressing these definitions. The authors of this paper are of the opinion that the form given in Table I is somewhat more intelligible to the average reader. Thus, luminance, in the CGS system of units, is defined as lumerg/sec/ω/cm². This, of course, is read as follows: lumergs per second per unit solid angle per square centimeter. In our opinion, the sequence of mathematical operations indicated by these formulas is unambiguous and somewhat preferable to the form used in reference 5. The letter symbols proposed in that reference (as in Table I) are in complete harmony with the usage recommended by the Sectional Committees Z10 and Z7 of the American Standards Association, and these letter symbols are entirely satisfactory in discussions not involving photographic concepts for which conflicting usages have been established and have stood for the last four or five decades.

Since this communication deals predominantly with photographic problems, we find it inadvisable to alter the letter symbols which have been used in this field for certain concepts. Thus, E has been firmly established in the field of scientific photographic literature as a symbol for *exposure*, while the symbol I has been used throughout the same period for the designation of *illuminance*. Thus, photographic exposure, E, is defined as the product of I (expressed in meter-candles) by *exposure time*, t (sec.). We feel that it is imperative for us to adhere to the usage of these letter symbols in this communication and we are faced therefore with the necessity of choosing other letter symbols for the designation of the photometric concepts, *illuminance* and *luminous intensity*. As noted later in the discussion, we will use I as the letter symbol for illuminance and, although we shall find little occasion to use the concept *luminous intensity*, where this is necessary we shall use the letter symbol C.

The approach to the exposure problem in this paper may be briefly set forth as follows: The total amount of light reaching a point at the earth's surface when the sun is at the zenith and the earth's atmosphere is *clear* will be taken as the reference point in terms of which to express the total amount of light available at various hours, months, and geographical latitudes. The modification of the amount of light available for photographic purposes from the ideal condition of *clear atmosphere* resulting from the following factors will then be considered separately: (a) atmospheric conditions other than a perfectly clear atmosphere, (b) scene structure, (c) local illumination conditions within the scene itself, (d) the directional viewing aspect, and (e) the spectral quality of sunlight and skylight.

II. EXPOSURE

Before discussing in detail the factors involved in the exposure problem, the more general fundamental relationships will be considered briefly.

A. The Definition of Exposure

The word "exposure" has been defined in a precise and quantitative manner as the product of the illuminance, I, on the photo-sensitive surface and the exposure time, t, during which this illuminance is permitted to act. This definition has been established not only by common usage in the literature of photography, but also by official action of national and international standardizing bodies. The relationship is shown formally as

$$\text{Exposure } (E) = I \cdot t. \qquad (1)$$

It is standard practice to express illuminance in meter-candles and time in seconds. Thus, the photometric unit in which exposure is expressed is the meter-candle-second (mcs).

B. Image Illuminance

For the purposes of "black-and-white" photography, the image on the focal plane of the camera can be specified completely in terms of the distribution of the illuminance on that plane. At any point this illuminance is attributable to two distinct sources: (a) the illuminance due to the light coming directly, by virtue of the refractive characteristics of the lens system, from a point on the object (scene), and (b) the illuminance due to "flare light." The term *flare light* is used to designate the illuminance which is distributed more or less uniformly over the focal plane. It is due to various causes, such as interreflections between the glass-air surfaces of the lens system, reflection from the interior surfaces of the lens mount, shutter blades, and diaphragm blades, and reflection from the interior surfaces of the camera body and bellows. In a sense this flare light is non-image-forming light but nevertheless must be considered as a part of the image which acts upon the negative material. Flare light is distinguished from localized concentrations of illuminance due to reflected or refracted light giving rise to "flare spots." The symbol I_i is used to denote the general aspect of image illuminance. The more specific aspects are indicated as follows:

I_{io} = image-forming illuminance coming directly from the object

I_{if} = non-image-forming illuminance or flare light

I_{it} = total image illuminance.

$$I_{it}=I_{io}+I_{if}. \quad (2)$$

The relation between object luminance, B_o, and image illuminance, I_{io}, for any lens system of specified characteristics is well known. Formulas giving this relationship have been published by several investigators among whom may be mentioned Nutting,[6] Moffitt,[7] Hardy and Perrin,[8] Goodwin,[9] and Jones and Condit.[10] The complete formula as given by Jones and Condit is:

$$I_{io}=B_o\cdot(F^2/4v^2f^2)\cdot\cos^4\theta\cdot H\cdot T_g\cdot 10.76, \quad (3)$$

in which the symbols have the following significance:

I_{io} = image illuminance (meter-candles)

B_o = object luminance (foot-lamberts)

F = focal length

v = image distance

d = diameter of opening in lens diaphragm

$f=F/d$, aperture ratio

θ = angle of image point off axis

T_g = lens transmittance as limited by losses due to reflection at glass-air surfaces and by absorption in the glass

H = lens transmittance at off-axis points as limited by vignetting.

Equation (3) may be rewritten in the form

$$I_{io}=(B_o/f^2)\cdot K, \quad (4)$$

where

$$K=(F^2/4v^2)\cdot\cos^4\theta\cdot H\cdot T_g\cdot 10.76. \quad (5)$$

By making assumptions as to the most probable average values of the factors which determine the value of K, as shown in (5), Jones and Condit[10] concluded that, for the purpose of obtaining a simplified expression, the value $K=1.25$ should be satisfactory. The assumptions[e] made apply specifically to conditions which exist particularly in the field of amateur photography where relatively small and compact cameras are in general use. Combining Eqs. (2) and (4) we obtain

$$I_{it}=(B_o/f^2)\cdot K+I_{if}. \quad (6)$$

Substituting this value of I_{it} in (1) it will be seen that

$$E_k=\left(\frac{B_o\cdot K}{f^2}+I_{if}\right)\cdot t, \quad (7)$$

where E_k is the exposure (in mcs) incident on the photographic material in the camera, where the image of a scene element having a luminance B_o is formed.

C. The Evaluation of Flare Light

To determine the exposure (E_k) incident on the negative material for specified values of B_o, f, and t, it is necessary to know I_{it}, which can only be obtained by knowing also how much flare light, I_{if}, is present. Some method of evaluating flare light is therefore necessary.

Instrumental methods for measuring I_{if} are available but they are not easily applicable to practical field work. The amount of flare light incident on the focal plane in a camera equipped with some specific lens system is not determined entirely by the characteristics of the camera-lens system, but is dependent in a large measure upon the distribution of luminance within the scene being photographed and in its environment. Laboratory measurements of the physical and optical characteristics of the lens system are of little use therefore for the evaluation of the amount of flare light present under a particular field condition.

However, by measuring I_{if}, for a large number of scenes with a particular camera equipment, a statistical average value can be obtained which is sufficiently valid for practical purposes when that equipment or others similar in physical and optical characteristics is being used. Moreover, by making a sufficiently large number of such measurements with various types of camera equipment, an average value applicable to a given general type of work may be established.

One method for evaluating flare light is illus-

[e] For a detailed discussion of these assumptions the reader is referred to the original publication.

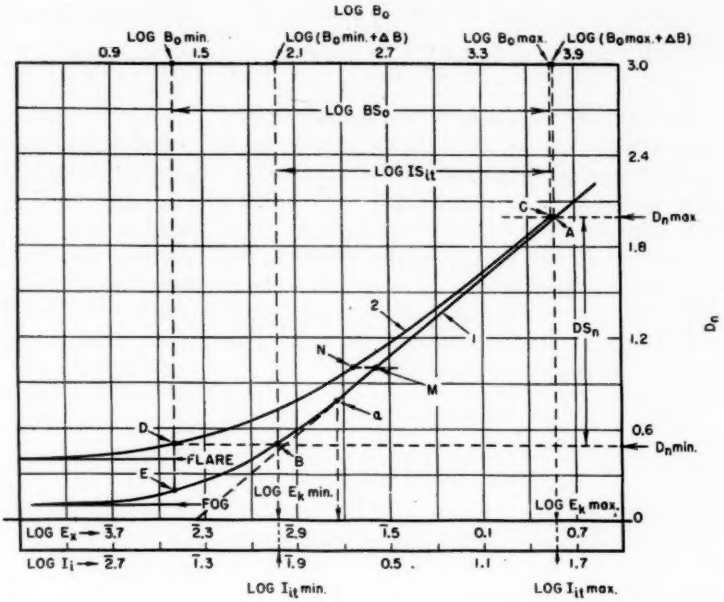


FIG. 1. Evaluation of flare factor, *FF*. Curve 1, the characteristic of the ***negative material***, $D_x = f(\log E_x)$. Curve 2, the characteristic of the ***negative***, $D_n = f(\log B_o)$.

trated in Fig. 1.[d] Using a camera having a lens system for which K is known, a photograph is made (with known values of t and f) of a scene in which the maximum luminance, B_o max, and the minimum luminance, B_o min, have been measured. The exposed negative material is then developed, along with an exposed sensitometric strip of the same negative material. From the density measurements made on this developed sensitometric strip, curve 1 (Fig. 1) is plotted, $D_x = f(\log E_x)$. The densities in the negative at the points where the maximum and minimum object luminances are rendered are also measured, thus giving values of minimum negative density, D_n min, and maximum negative density, D_n max. These values are located on the density scale, as indicated in Fig. 1, and horizontal lines through these points locate the points A and B on the D_x-$\log E_x$ curve. The points A and B lie at the extremities of the ***used portion*** of the ***negative material characteristic.*** Perpendiculars dropped from these points to the $\log E_x$ axis determine the values of the maximum and minimum exposure incident on the negative material in the camera, $\log E_k$ max and $\log E_k$ min. The exposure time, t, used in making the negative being known, values of I_{it} max and I_{it} min may be obtained.

The curve BA is now a graphic representation of

$$D_n = f(\log I_{it}).$$

The illuminance scale of the image, IS_{it}, is given by

$$IS_{it} = I_{it}\ \max / I_{it}\ \min. \qquad (8)$$

[d] If the spectral sensitivity of the photographic material on which the negative is made is not the same as that of the human eye, this method may be subject to some error. Measurements of object luminance are made by a visual method and hence are evaluated in terms of the spectral sensitivity of the human eye. If the flare light has a spectral composition differing appreciably from that used in the sensitometric determination of Curve 1, or if the light reflected from the object areas of maximum and minimum luminance differ markedly in spectral composition from the sensitometric illuminant, some error in the value of flare factor determined by this method may occur. While the existence of such conditions may result in some uncertainty of the flare factor for a single scene, the error in the average flare factor for a large number of scenes is probably sufficiently small to be negligible. This follows because it is likely that the difference between the visually measured luminance scale, BS_o, and the photographically determined illuminance scale, IS_{it}, is in some cases too great and in other cases too small. The existing evidence indicates that for a large number of different scenes taken under a wide variety of conditions and locations the positive errors will almost exactly balance the negative errors. However, it seems desirable, if possible, to eliminate the occurrence of such errors in the measurements made on a single scene. Our present practice, therefore, in the continuation of work of this type is to use a negative material with a filter having selective absorptance characteristics such that the effective spectral sensitivity of the photographic material-filter combination is identical to that of the average human eye.

The luminance scale, BS_o, of the object (scene) is given by

$$BS_o = B_o \max / B_o \min. \quad (9)$$

If flare light is present in the camera during the making of the negative, the illuminance scale of the image is always less than the luminance scale of the object. The magnitude of the difference between these two scales is a measure of the amount of flare light. For many purposes the evaluation of flare light in terms of the ratio of these two scale values is most significant and quite sufficient. The *flare factor*, *FF*, defined in this manner is given by

$$FF = BS_o / IS_{it}. \quad (10)$$

The flare factor, however, does not yield directly a value of I_{if}, which is necessary for many purposes.

The presence of flare light in the camera may be regarded as equivalent, in its effect upon the total image illuminance, I_{it} and IS_{it}, to the addition of a *constant luminance increment*, ΔB, to all of the actual luminances in the scene. If this hypothetical scene, the luminances of which are obtained by adding the constant ΔB to all of the luminance values found in the real scene, is photographed with a camera-lens system having zero flare, the distribution of illuminance on the focal plane of the camera will be the same as if the real scene were photographed with a camera-lens system having a flare factor as measured under the conditions of the actual scene and its environment. The value of this constant luminance increment is given by

$$\Delta B = \frac{(E_k \min \cdot B_o \max) - (E_k \max \cdot B_o \min)}{E_k \max - E_k \min}. \quad (11)$$

Since the object luminance increment, ΔB, is imaged by the same lens system as the actual object luminances, an expression analogous to (4) yields the evaluation of I_{if}. Thus,

$$I_{if} = (\Delta B / f^2) \cdot K. \quad (12)$$

Similarly, it is evident that the total illuminance incident on the focal plane due to the image-forming and the non-image-forming light is given by

$$I_{it} = [(B_o + \Delta B)/f^2] \cdot K. \quad (13)$$

Inserting this value of I_{it} in (1) gives

$$E_k = \frac{B_o + \Delta B}{f^2} \cdot K \cdot t. \quad (14)$$

Having determined the value of ΔB, it is possible to establish at the top of Fig. 1 a logarithmic scale of object luminance, B_o. A perpendicular through the point A must read on the scale the value of $\log(B_o \max + \Delta B)$. Likewise, a perpendicular through B must indicate on this scale the value of $\log(B_o \min + \Delta B)$.

Furthermore, a second curve, one showing the relation between $\log B_o$ and D_n, can now be plotted. The point C is located at the left of A (at the same D_n value) by a distance which is given by

$$\log \frac{B_o \max + \Delta B}{B_o \max}. \quad (15)$$

The point D is located to the left of B at a distance which is given by

$$\log \frac{B_o \min + \Delta B}{B_o \min}. \quad (16)$$

These two points lie at the extremities of the *characteristic curve of the negative*, $D_n = f(\log B_o)$, as indicated in curve 2.

For *any* point, such as M on curve 1, a corresponding value of $\log(B_o + \Delta B)$ may be read from the scale at the top of the figure. By using the known and constant value of ΔB, $\log B_o$, corresponding to the D_n value, the point N may be found, thus locating the point N on a horizontal line to the left of point M. Point N must lie on the curve DC. In this way any desired number of points can be established for the precise determination of the entire DC curve. When it is necessary to determine the magnitude of a negative density difference, ΔD_n, corresponding to some specified luminance difference, $\Delta \log B_o$, curve 2 must be used. Tone reproduction theory indicates that the useful portion of the characteristic D-log$E$ curve is limited in the extreme low and high density regions when the slope of the curve falls below some value which is insufficient to reproduce satisfactorily luminance differences either in the shadow or highlight region of the scene. For the establishment of such limiting gradient points, curve 2 is indispensable.

For many scenes and cameras, the *average* value of ΔB is of little significance, since the same value of ΔB, for scenes differing widely in luminance, may produce very different effects on the shortening of the illuminance scale of the image, on the reduction of luminance contrast in the image, and on the exposure incident on the negative material. On the other hand, an *average* value of the flare factor, FF, is significant, since it expresses I_{if} in the form a ratio and hence is not dependent upon the absolute values of B_o. Equation (14) is adequate for all cases where a specific value of ΔB can be obtained, but when it is desired to set up an equation which will take into account the *average* flare-light conditions for a wide range of scene types and scene environments, the ΔB in Eq. (14) must be replaced by an equivalent expression of flare light in terms of flare factor.

Inspection of Fig. 1 shows that the log of the flare factor is given by subtracting the length of the line AC from that of BD. This difference is given by

$$\log FF = \log \frac{B_o \min + \Delta B}{B_o \min} - \log \frac{B_o \max + AB}{B_o \max}. \quad (17)$$

The expression obtained by solving (17) for ΔB contains both B_o min and B_o max and is somewhat too complex for convenient use in a simplified equation for E_k. A very close approximation is obtained by neglecting the last term of Eq. (17), that is,

$$\log \frac{B_o \max + AB}{B_o \max}. \quad (15)$$

In general, ΔB is very small compared with B_o max. Hence, the magnitude of the term (15) is always small compared with that of the first term,

$$\log \frac{B_o \min + \Delta B}{B_o \min}. \quad (16)$$

Jones and Condit[10] have reported values of ΔB for some 126 exterior scenes on which luminance measurements were made and the results analyzed as illustrated in Fig. 1. The maximum separation between the points A and C found by them is 0.02. By neglecting the term (15) in Eq. (17), the points A and C become coincident. Their assumption that these points are coincident leads to a very small error, never exceeding 5 percent. If, therefore, this approximation be accepted as precise enough for practical purposes, the flare factor is given by

$$\log FF = \log \frac{B_o \min + \Delta B}{B_o \min}. \quad (18)$$

Solving for ΔB,

$$\Delta B = B_o \min(FF - 1).$$

Substituting this value of ΔB in (13) gives

$$I_{it} = \frac{B_o + B_o \min(FF-1)}{f^2} \cdot K. \quad (13a)$$

Combining (13a) and (1),

$$E_k = \frac{B_o + B_o \min(FF-1)}{f^2} \cdot Kt. \quad (14a)$$

If B_o is equal to B_o min, then

$$I_{it} \min = \frac{B_o \min \cdot FF}{f^2} \cdot K \quad (13b)$$

and

$$E_k \min = \frac{B_o \min \cdot FF}{f^2} \cdot Kt. \quad (14b)$$

The value of FF and of ΔB obtained by the graphic construction illustrated in Fig. 1 and by the analytical expressions derived is not dependent upon any knowledge of or assumptions as to the numerical value of t, f, or K. Hence, if t and f are known, the value of K can be computed from (14b). This method, which is quite convenient for use, yields satisfactory results for the determination of K, and serves as a convenient method of studying the way in which K depends upon such factors as the change in barrel vignetting as the diaphragm is closed down and the relationship between K and the off-axis distance in the image.

D. Camera Exposure, *CE*

The word "exposure" is used in verbal discussions and in the literature of photographic problems to express at least two different ideas. Usually, the context indicates which of the possible meanings is intended. Confusion, however, does sometimes arise, and it seems undesirable in a technical discussion to define one word in more than one way.

One definition of exposure has already been given in Eq. (1). The other idea which is commonly associated with the word is that implied by such terms as "exposure table," "exposure guide," "exposure calculator," and "exposure meter." The same connotation is implied in the question, "How much exposure should be given under these conditions?" or in the statement, "It is necessary to give more exposure on a dull day than on a bright one." These usages of the word do not carry the same meaning as when used in the statement, "The photographic material received an exposure of 50 mcs."

In making a photograph of a scene, the operator endeavors to adjust the magnitude of the exposure administered to the photographic material in the camera to such a value that an excellent print can be made from the resultant negative. The amount of exposure for this purpose is dependent only on the negative material and is not determined by the luminance of the scene. Thus, the required *exposure* is fixed once the negative material has been chosen. If negatives of a uniform quality are desired, then the same exposure must be given to the negative material, regardless of the luminance characteristics of the scene. The thing that the operator must do is to adjust the camera elements which control the I and t factors of exposure so that the exposure required is administered to the negative material. This is done by setting the shutter which controls the exposure time, t, and the diaphragm which controls the illuminance, I, incident on the focal plane.

Equation (14) may be rewritten in the form

$$t/f^2 = E_k/(B_o+\Delta B)\cdot K. \qquad (14c)$$

The only function of an "exposure" guide, table, calculator, or meter (aside from providing convenient aids for performing simple computational operations) is to enable the operator to determine values of t and f which will result in the application of the correct exposure to the negative material. The two factors t and f are the only ones appearing in (14c) which are controlled by adjustments of elements of the camera and hence they represent the camera's entire contribution to the control of exposure. It has been suggested that the term *camera exposure* instead of "exposure" be used in referring to this value; thus,

$$\text{Camera exposure} = t/f^2.$$

Since t is expressed in seconds and f is a pure numeric, the dimension of camera exposure is time and is expressible in seconds. ***Camera exposure, CE,*** therefore becomes equal to exposure time, t, when $f=1.0$.

E. The Speed of Photographic Negative Materials

Curve 1 (Fig. 1) constitutes a complete specification of the sensitivity of that photographic material to radiant energy of a specific quality. The quality (spectral composition) which has been adopted by international agreement for sensitometric measurements on negative materials is that emitted by a specified tungsten lamp-filter combination and corresponds approximately to the quality of mean noon sunlight at Washington, D. C. It must be remembered that if the quality of radiant energy reaching the photographic material in practical work differs from that of the standard just mentioned, a correction must be made which involves a knowledge of the photographic efficiency of the radiant energy reaching the photographic material in the camera.

Curve 1 (in Fig. 1) shows directly the density produced by any value of exposure. If it is desired to render *any* selected value of object luminance at *any* selected point, a, on the negative material characteristic, it is only necessary to use Eq. (14c) which gives the required value of camera exposure.

$$t/f^2 = \frac{E_a}{(B_o+\Delta B)\cdot K}, \qquad (14d)$$

where E_a is the exposure corresponding to the selected point, a. This Eq. (14d) represents a complete solution of the exposure problem in its most generalized form.

For practical purposes, the sensitivity of a photographic material should be expressed in terms of a single number rather than in terms of the entire function, $D_x=f(\log E_x)$, so some criterion must be chosen. This criterion must be unique and, in addition, should be meaningful. Thus, the point a on curve 1 can be given satisfactory uniqueness by specifying its density. It is, however, no more meaningful than any other point on this curve chosen at random. If the speed of a photographic material is to be a reliable guide for its use in the camera, then the significance of a speed criterion must be appraised in terms of the infallibility with which its use leads to the desired result. This result may conceivably be a negative of some ideal quality or negatives of a uniform characteristic, or possibly a positive meeting certain specifications. In any negative-positive process where the negative is only a means to an end, the most desirable result of a photographic exposure is a negative from which a print of excellent quality can be made. Since most high quality negative materials have an exposure scale which is large compared with the illuminance scale of the image, a wide variation in camera exposure may yield negatives meeting this specification. Hence, the criterion as just defined lacks satisfactory uniqueness. It is generally agreed, therefore, that speed should be expressed in terms of the *minimum* exposure which will produce a negative from which an excellent print can be made. Speed can actually be evaluated in this manner by a statistical psychophysical method[11] involving no sensitometric operations. Such a method, however, is too complicated and laborious to be of practical value.

A sensitometric method has been found which yields results in close harmony with those derived from the statistical psychophysical determinations. This method leads to the establishment of a speed criterion which is both unique and significant. It has been shown[10–12] that the conditions of minimum exposure and excellent print quality are obtained when the lowest luminance of the scene, B_o min, is rendered at a point on the characteristic D_x-$\log E_x$ curve where the slope or gradient, $dD/d\log E$, is a constant fractional part of the average slope $\bar{G}$ of a portion of the curve extending a distance $\Delta\log E=1.50$ in the direction of increasing exposure from the point mentioned. This criterion of effective camera speed is referred to as a *fractional gradient criterion.*[a]

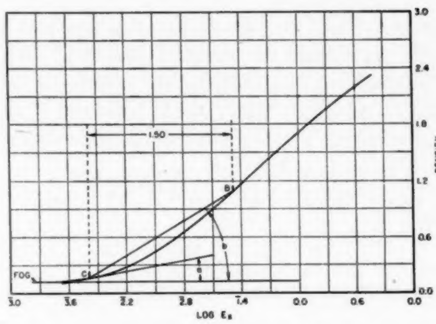


Fig. 2. Graphic representation of the fractional gradient criterion.

The definition of this criterion is illustrated graphically in Fig. 2. The gradient, $dD/d\log E$, of the curve at the point C is given by the tangent of the angle a, this angle being that included between the horizontal base line and a straight line drawn through the point C tangent to the characteristic curve. By measuring a distance of 1.50 on the $\log E_x$ scale from C, the point B is established on the curve. The average slope, $\bar{G}$, of the part of the curve lying between C and B is that of a straight line drawn between the two points. The angle included between this straight line and the horizontal base line is b. Tan $b=\bar{G}$. When the gradient at point C is equal to $0.3\bar{G}$, the exposure, $E_{0.3\bar{G}}$, determines the fractional gradient speed for this material. Speed is then expressed as the reciprocal of $E_{0.3\bar{G}}$.

$$\text{Speed } (S)=1/E_{0.3\bar{G}}. \tag{19}$$

Other criteria of speed have been proposed and used during the past three or four decades. These include: inertia, on which Hurter and Driffield based their values of speed; fixed density criteria; and fixed gradient criteria. It appears now that all three of these are unsatisfactory criteria of effective camera speed because they lack significance as appraised by the yardsticks of excellent print quality and minimum camera exposure. At present there seems to be quite general agreement

[a] The American Standards Association has recently issued an American Standard (ASA Z38.2.1–1946) covering a method for the measurement and specification of the speed of photographic negative materials. The criterion of speed used in this standard is the "fractional gradient criterion" as defined in this paragraph.

that the fractional gradient criterion just discussed represents the most satisfactory solution of the problem of specifying the effective camera speed of negative materials in terms of a single number which can be determined sensitometrically with ease and precision.

Now, combining (19) and (14b) and solving for t,

$$t = f^2/B_o \min \cdot FF \cdot K \cdot S, \quad (20)$$

or solving for camera exposure,

$$t/f^2 = 1/B_o \min \cdot FF \cdot K \cdot S. \quad (20a)$$

Of the factors appearing in the right-hand member of (20a), all but one are known or can be determined with certainty sufficient for all practical purposes.[f] Sensitometric measurements yield values of S of great reliability. The value of K for a specific camera-lens combination can be determined with high precision and even for a large group of camera-lens combinations an average value of sufficient certainty can be established. A statistical average value of flare factor for a large number of exterior scenes varying widely in brightness characteristics, made with a camera-lens structure of known characteristics, has been determined by Jones and Condit. This value is 2.40. The camera-lens system used by them is of a type known to be more free from flare light than the average small compact camera used largely in the amateur field. From laboratory measurements (supported by some field measurements) they have concluded that for the amateur field an average flare factor of 4.0 is probable. For the commercial field where larger cameras are usually used, the average flare factor should probably be somewhat less than 4.0.

The only unknown factor in the right-hand member of Eq. (20a) is that which defines the luminance characteristic of the object. In practical field work, it varies over a great range of values. The luminance of any surface as seen by an observer depends upon two factors: (a) the *illuminance* on the surface and (b) the *reflectance* of that surface, evaluated with respect to the direction of incidence of the illuminance and the viewing angle. These two factors must be treated separately, and in the next section we shall turn our attention to the evaluation of the amount of light reaching the earth's surface from the sun which is the only significant source of natural illumination during daylight hours.

[f] The validity of Eq. (20a) depends upon the assumption that the spectral energy composition of the light reflected from the scene element of minimum luminance (B_o min) is identical to that used in the evaluation of S. The consequences of departures from this assumed condition will be discussed in Part II of this paper to be published later.

III. SOLAR IRRADIANCE AND ILLUMINANCE

The light which reaches the earth's surface from the sun consists of two components: *direct sunlight* and *skylight*. The former term designates the light that comes directly from the sun, while the latter component is solar radiant energy scattered by the earth's atmosphere. Quite apart from the differences in intensity and quality, these two sources behave very differently when considered photographically. The sun's disk subtends a small angle, 0.5 degree, at a point on the earth's surface, and direct sunlight, therefore, produces strong cast shadows. On the other hand, the sky is a source of large angular dimensions. At any point on the earth's surface the sky hemisphere subtends a solid angle of 2π steradians. Skylight, therefore, forms no cast shadows and, because of its great angular distribution, tends to illuminate scene elements which receive no illuminance by direct sunlight except perhaps that reflected from nearby terrestrial objects.

The earth's atmosphere consists of a gaseous envelope approximately 60 miles thick. While, of course, the upper boundary of the atmosphere is not sharply defined, less than one-millionth of its mass lies above this 60-mile height. It is composed largely of oxygen, 20 percent, and nitrogen, 78 percent. The remainder is made up of small amounts of rare gases (helium, neon, argon, krypton, xenon), carbon dioxide, ozone, and water vapor. The amount of the last component varies from time to time between wide limits. Even when the atmosphere is not contaminated by the presence of *condensed* water vapor, dust particles, smoke, etc., the direct sunlight is scattered by the molecules of the gases and vapors. Rayleigh has shown that the amount of this scatter is proportional to the fourth power of the wave-length of the radiant energy. This gives rise to an appreciable scatter of the shorter wave-lengths of visible radiant energy but relatively little scattering of the longer wave-lengths. The sky under these conditions has a deep blue color and a relatively low luminance. When condensed water vapor, dust, smoke, etc., are present, the particles are usually of sufficient size so that the amount of scattering is approximately the same for all wave-lengths of visible radiant energy. Under these conditions, the color of the sky varies from light bluish gray through white to dark gray, depending upon the amount of scattering material present. Under some conditions, the particle size of the condensed water vapor, dust, etc., may be small enough to produce more scattering of the shorter wave-lengths of visible radiant energy. This is the cause of the well-known blue haze which is so frequently seen in hilly and mountainous regions.

A. Direct Sunlight, Clear Atmosphere

The term *clear atmosphere* is defined here as an atmosphere containing the normal or average proportions of the gaseous and vaporous components but is sensibly free from condensed water vapor in the form of liquid water droplets

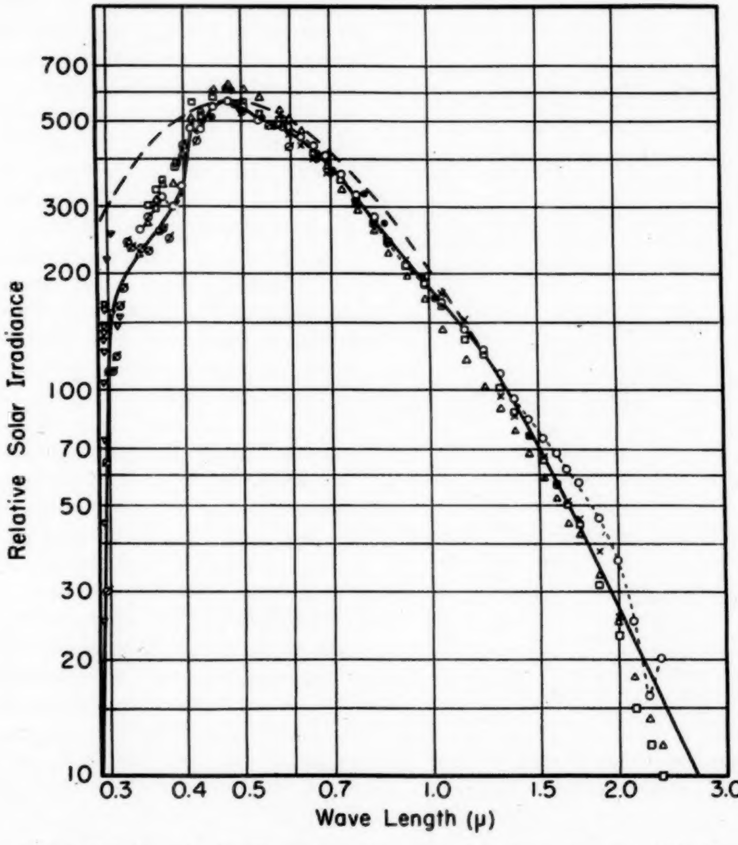


FIG. 3. Relative solar irradiance on a plane normal to the sun's rays outside the earth's atmosphere (from Moon, ref. 3, Fig. 1).

△ Fabry and Buisson
ϕ Pettit
● Wilsing
Δ 1903–1910 (Smithsonian Institution)
× 1903–1910 (omitting quartz results) (Smithsonian Institution)
□ 1916–1918 (Smithsonian Institution)
0--0 1920–1922 (Smithsonian Institution)
— — Blackbody, 6000°K
—— Proposed standard curve

or ice crystals and of solid particles such as dust, smoke, etc. The amount of radiant energy and, consequently, of light, arriving at a point on the earth's surface in the form of direct sunlight, varies widely. According to Kimball[13] there are six principal causes of this variation. They are as follows:

"1. Variations in the amount of heat energy radiated from the sun, the intensity of which at the earth's mean solar distance is called the solar constant. These are small, apparently non-periodic variations, and need not be considered in this investigation.

"2. Variations in the earth's solar distance. Since the radiation intensity varies inversely as the square of this distance, we would expect it to be 7 percent more intense with the earth in perihelion early in January than with the earth in aphelion early in July.

"3. Amount of water vapor in the atmosphere. In general, this decreases with latitude, altitude, and distance from the ocean, and increases with temperature.

"4. Dustiness or haziness of the atmosphere. This appears to be closely related to 3.

"5. The zenith distance of the sun, which is easily computed when we know the latitude of the place of observation, the sun's declination, and the apparent, or true solar time.

"6. Obviously, the irradiation will, in general, increase with altitude."

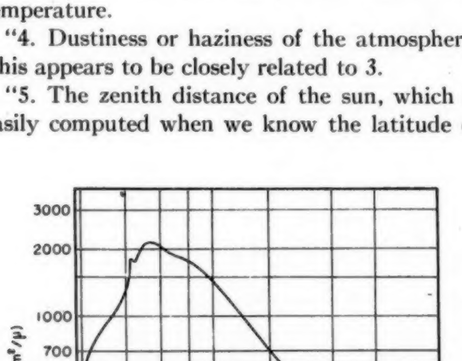


FIG. 4. Proposed standard solar irradiance curve (watts/m^2/μ), on a plane normal to the sun's rays, outside the earth's atmosphere and at the mean solar distance (from Moon ref. 3, Fig. 2).

Of these causes the variations due to 1, 2, 3, and 6 are small compared with those mentioned in 4 and 5. For the time being, a *clear atmosphere* will be assumed. Hence, 4 may be neglected, leaving 5 as the controlling factor to be discussed in this section.

1. The solar constant.—Moon[3] begins his summary of the available data bearing on various aspects of direct sunlight by giving a weighted mean value for the solar constant. This is a measure of the amount of radiant energy from the sun incident per unit area on a plane perpendicular to the sun's rays (normal plane) just *outside* the earth's atmosphere. The value given is 1322 watts per square meter. The source material used in deriving this value is indicated by the literature references 14 to 18, inclusive.

2. Spectral distribution of radiant energy.—In Fig. 3 is shown Moon's treatment of the data on this subject. The source material is drawn from literature references 19 to 23, inclusive. In discussing this figure, Moon states:

"The data of Fig. 1 [Fig. 3 of this communication] are in arbitrary units, and thus it is permissible to move any set of data up and down on the diagram. The Smithsonian results were plotted directly from the table given by Abbot, Fowle and Aldrich,[8] [our reference 20] while the other sets of points were shifted vertically to give the best match. The blackbody curve[11] [the dashed curve, our reference 23] was adjusted so that its maximum was approximately equal to the maximum of the 1920–22 curve.

"The best result of the Smithsonian Institution is usually believed to be[8] [our reference 20] the weighted mean of the 1920 and 1922 results, which is shown by the circles and the dotted curve of Fig. 1 [our Fig. 3]."

Moon has thus given due weight and consideration to the available data. In Fig. 4 is shown Moon's proposed standard curve, the ordinates of which are now given in absolute units, watts per square meter per micron.[g] This

[g] The significance of the "per micron" part of this phrase may be somewhat obscure to some readers. The ordinate value corresponding to any *point* on the curve in Fig. 4

standard curve of course refers to sunlight outside the earth's atmosphere.

3. Losses due to absorption and scattering in the earth's atmosphere.—The next step in Moon's analysis deals with a quantitative evaluation of the radiant energy which is lost because of scattering and absorption by the earth's atmosphere. The source material used for this purpose is indicated by literature references 24 to 34, inclusive. The scattering by gas molecules is shown to be in good accord with Rayleigh's fourth-power law. Absorption is due chiefly to ozone, in the spectral region between 300 and 400 mμ, and to water vapor, the absorption bands of which fall largely in the infra-red region, although there is a small absorption band at 690 mμ. These absorption effects lie largely outside of the visible spectrum and hence are of little concern in the field of photography with which this discussion deals. It must not be forgotten, however, that the absorption due to water vapor is of considerable importance when photography in the near infra-red region is undertaken. Moon also applies a correction for loss by scattering due to "water vapor." From Fowle's data he concluded that between 400 and 1000 mμ this scattering is proportional to the minus second power of the wave-length. Such scattering cannot be due to water in the gaseous state, but must be ascribed to water condensed into particles of an average size such as to give the minus second power relationship. The scattering effect of atmospheric dust is also discussed on page 593 and Fig. 3 of reference 3.

4. Proposed standard solar irradiation curves.—By applying these correction values for losses by scattering and absorption to the curve shown in Fig. 4, Moon finally arrived at a series of spectral energy distribution curves, covering the wave-

indicates the radiant flux in watts per square centimeter, its absolute value depending upon the width of the spectral band used in its evaluation. Any single point on the curve in Fig. 4, of course, strictly speaking, corresponds to a spectral band of infinitesimal width, for which, of course, the value of radiant flux per unit area is also infinitesimal. Any finite value of flux per unit area applies to a spectral band width which has a finite value. It is customary, therefore, to express the absolute magnitude of radiant flux per unit area in terms of a spectral band having unit wave-length width. When the abscissa values are expressed in microns, it is customary to express the absolute value of radiant flux in terms of a spectral band having a width, Δλ, of one micron. Likewise, if the abscissa values are expressed in terms of millimicrons (mμ) it is customary to express the absolute value of radiant flux per unit area in terms of a spectral band having a width, Δλ, of 1 mμ.

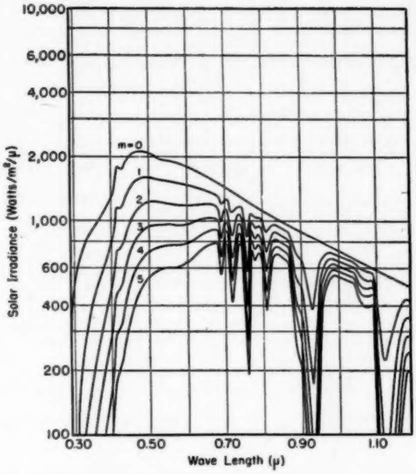


FIG. 5. Proposed standard solar irradiance curves (watts/m^2/μ), on a plane normal to the sun's rays, outside the earth's atmosphere ($m=0$) and at the earth's surface for air masses of 1, 2, 3, 4, and 5. (Moon, ref. 3, Fig. 10.) Conditions: atmospheric pressure = 760 mm, precipitable water = 20 mm, dust = 300 particles/cm^3, ozone = 2.8 mm.

length band from 300 mμ to 2000 mμ, which show that spectral distribution of direct sunlight incident at the earth's surface after having traversed various thicknesses of the earth's atmosphere. The length of path through the earth's atmosphere which rays from the sun must traverse before reaching a point on the earth's surface is usually expressed in terms of air mass. When the sun is at the zenith, the air mass is taken as unity, 1.0. The air mass for any zenith distance other than zero is given approximately by the secant of the sun's zenith distance. When more precise values are required, a correction must be applied which takes into account the curvature of the earth's surface and the refraction or bending of the sun's rays as they traverse the earth's atmosphere. This bending results from the change in atmospheric density (refractive index) with altitude. The formula which represents this desired correction most precisely is that due to Benporad:[35]

$$m = \frac{\text{atm. refr. in seconds}}{58.36 \cdot \sin z}, \tag{21}$$

where m is the air mass, and z is zenith distance.

The standard curves proposed by Moon for the

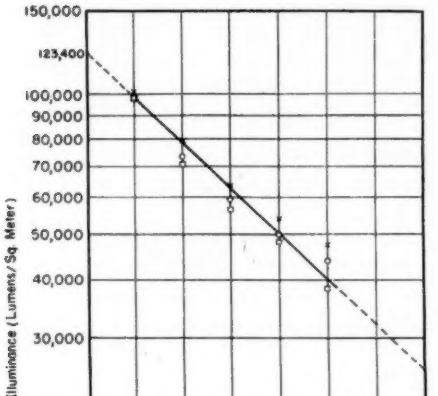


FIG. 6. Direct solar illuminance (lumens/m^2) at the earth's surface on a plane normal to the sun's rays. (From Moon, ref. 3, Fig. 15.) The straight solid and extrapolated dashed curve computed by Moon from Fig. 5. Kimball's directly determined experimental values are represented by the plotted points {○ May, × April, □ June}.

spectral region 300 mμ to 1200 mμ are shown in Fig. 5. These are for various air masses as indicated by the number near each of the curves. These curves give the radiant energy in watts per square meter per micron incident at the earth's surface (or just outside the earth's atmosphere, for $m=0$), the evaluation being on a plane normal to the sun's rays. They are applicable to a point at sea level with some condensed water present and a relatively small amount of dust. Actually, they take into account the absorption due to the presence of a certain amount of ozone and of water vapor, but since the effect of these atmospheric components is largely in the ultraviolet and infra-red, they are not important here. The effect of scattering can be seen clearly by noting the change in the shape of the curves for increasing air mass, $m=0$, $m=1$, $\cdots m=5$.

5. Proposed standard solar illuminance curves.—By using the international standard luminosity data for the average normal human eye and a value of 621 lumens per watt[h] for the maximum luminous efficiency of radiant energy (luminous efficiency for $\lambda=555$ mμ), Moon computed the relationship between illuminance, in lumens per square meter, and air mass, as shown in Fig. 6. This illuminance is that on a plane perpendicular to the direction of the sun's rays, the *normal plane*. These computed data are represented by the solid curve which is seen to be a straight line and has been extrapolated by the dashed line to include the air mass, $m=7$. The plotted points shown in Fig. 6 are derived from direct experimental measurements by Kimball who obtained a conversion factor to permit the computation of photometric values from the *measured* radiometric values. Experimental values vary somewhat with the month of the year, but the average is in good agreement with Moon's computed and proposed standard curve for this relationship. These measured illuminance values for high air mass values are greater than those derived by computation. In concluding his paper, Moon makes some remarks which are of sufficient interest to warrant repetition at this point. They are as follows:

"A wealth of data on sunlight has been obtained by astrophysicists, meteorologists, and others, but this information has been scattered through the literature and has not been generally available to engineers. The present paper correlates some of the data and specifies a proposed standard spectral-distribution curve for sunlight outside the atmosphere. Methods are given also for the calculation of the spectral irradiation curve for any elevation above sea level and for any air mass, and these methods lead to proposed standard curves to be used in engineering calculations dealing with direct sunlight at sea level.

"These curves are checked against independent data on total irradiation, ultraviolet irradiation, illumination, and color temperature. In all cases, the agreement between calculated and experimental results gives confidence in the validity of the proposed curves for various engineering applications."

The present authors are convinced that Moon's treatment of the data relative to direct sunlight

[h] A somewhat higher value, 650 lumens per watt, is now generally accepted. The use of this new value increases the illuminance due to direct sunlight by 4.7 percent compared with that computed by Moon.

is the best that has thus far been published. They feel it can be safely taken as a sound and firm foundation on which to base exposure recommendations. The thoroughness with which the literature was searched by Moon and the excellent judgment exercised in the averaging and weighting of data from all sources deserves the appreciation of those interested in a precise and reliable evaluation of the illuminance attributed to direct sunlight. Certainly this constitutes a service of great value to the photographic field.

The photometric unit used thus far in the expression of illuminance is lumens per square meter. In this country it is a more usual practice to express illuminance in terms of a different unit, the foot-candle: 1 lumen per square meter =0.0929 foot-candle. Moreover, as will be seen later, it is more convenient to deal with solar illuminance as a function of solar altitude rather than of air mass. Moon's computed data contained in the solid curve of Fig. 6 have been corrected by using the newer value of 650 lumens per watt and replotted as the heavy curve, A, in Fig. 7. To facilitate the comparison of his data with experimental results by other observers, the illumination on a horizontal plane, I_h, has been computed by using the simple expression

$$I_h = I_n \cdot \sin h, \tag{22}$$

where I_n is the illuminance on the normal plane (as shown in Fig. 6) and h is the solar altitude.

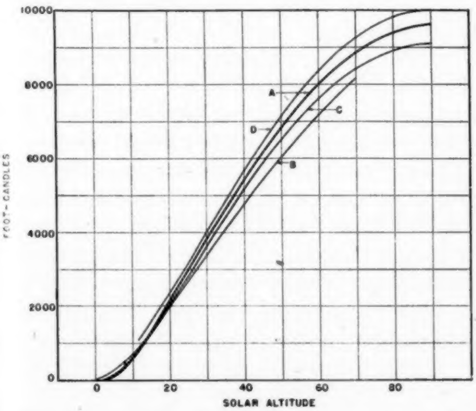


FIG. 7. Illuminance (foot-candles) due to direct sunlight on the horizontal plane, at the earth's surface, plotted as a function of solar altitude.

An extensive investigation was carried out by Kunerth and Miller.[36] Their values of illuminance on the horizontal plane as a function of solar altitude are shown in Curve B, Fig. 7. These values are the averages of observations made on 33 days scattered over a period of about 19 months, beginning November, 1929. The observation station was located on top of a three-story building in Ames, Iowa.[1] "Observations were made every hour on perfectly clear days only. . . . The morning and afternoon readings at the same altitude of the sun for the sun and sky combined were found to be very nearly the same and were averaged. For the sky alone at the same altitude of the sun, the readings were also averaged, although the afternoon readings were somewhat higher than the morning readings. An appreciable variation was found between different days even though they could be classed as perfectly clear, ***presumably because the condition of the atmosphere varies considerably even though not noticeable to the eye.*** [The italics are ours.]

"Since readings were taken on perfectly clear days only, long periods sometimes elapsed between readings, and yet the 33 days on which we were successful in obtaining readings all day were fairly well scattered over a period of 19 months."

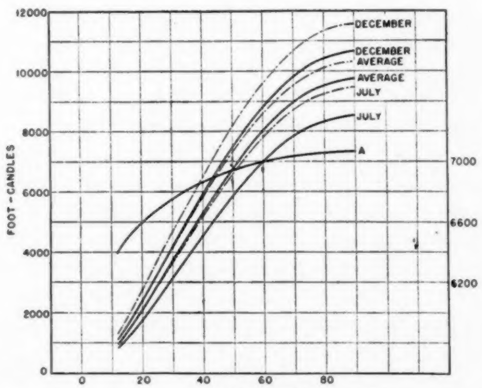


FIG. 8. Illuminance (foot-candles) due to direct sunlight on the horizontal plane plotted as a function of solar altitude. Solid curves for Washington, D. C., dashed curves for Lincoln, Nebr. Curve A "illumination equivalents" (foot-candles, right-hand ordinate) of 1 gram calorie per minute per cm² of solar radiation.

[1] Latitude 42°N, longitude 93½°W, and elevation 1000 feet.

It will be observed that the illuminance on the horizontal plane due to direct sunlight measured by these workers tends to be somewhat lower than those computed by Moon. Presumably, the total radiant energy values which Moon considers to be most authentic represent the condition when there is very little *condensed* water vapor in the atmosphere. Hence, it is possible that the lower values found by Kunerth and Miller may be accounted for by the presence of small amounts, undetectable to the eye, of condensed water vapor, although it does not appear likely that there could have been very much of such condensation because of the care which these workers used in selecting perfectly clear days.

Elvegård and Sjöstedt[37] have presented some further valuable information based upon measurements made by Aurén[38] in Sweden and by Lunelund[39] in Finland. After considering the values previously published by Kimball and Hand[40, 41] and by Kunerth and Miller (loc. cit.), Elvegård and Sjöstedt apparently preferred to use those of Aurén and Lunelund for the following stated reason: "Seeing that the results obtained by these investigators (Aurén and Lunelund) are based on the average values of an immensely large number of single measurements made in the course of several years and at different places in Scandinavia, they may be considered to be highly reliable." These data are represented by Curve *C* in Fig. 7. It will be seen that they fall somewhat lower than Moon's computed values but a little higher than those of Kunerth and Miller.

About 1900, the U. S. Weather Bureau began to collect information on the amount of solar radiant energy received at the earth's surface. Since that time they have published voluminous reports containing the results of their measurements. While at times they have made direct measurements of the illuminance due to the sun and sky, by far the greater part of their data relates to the radiant energy, in terms of gram calories per minute per square centimeter, received at the earth's surface from the sun and from the sky. That coming directly from the sun is designated by them as "solar radiation." These measurements have been made with high precision pyroheliometers at several stations scattered throughout the United States. Such measurements have been carried on systematically and continuously at these stations from approximately 1910 or 1915 to the present time. In order to convert these measurements of radiant energy into illuminance values, they have, from time to time, made simultaneous radiant energy and luminous energy measurements, thus obtaining "illumination equivalents of solar energy."

The first series of these measurements, made in 1919, were subject to some error arising from the use of an uncompensated illuminometer test plate. During the years 1921 and 1922 this work was repeated at the station near Washington, D. C., using a compensated illuminometer test plate which had been proved by exhaustive laboratory measurements to give correct results. This extensive piece of work thus provides a method for converting the "solar radiation" measurements at Washington, D. C., to illuminance values. In 1919 Kimball[13] published a summary of the measurements up to that time but, because of the doubtful quality of the "illumination equivalents" then available, their computed values of illuminance are subject to some question.

In 1922 Kimball and Hand[41] published a second summary of the Weather Bureau measurements on "solar radiation" up to that time and in this case the new "illumination equivalents" were used, giving values of illuminance which are subject to little doubt. In this particular communication, the information given relates specifically to one locality, namely, latitude 42°N with cloudless sky (east of the Mississippi River).

In 1937 Hand[27] published another summary of all the measurements made at several of the U. S. Weather Bureau stations. In this case no values of luminous intensity are given but, by using the "illumination equivalents" mentioned, it is possible to convert his values of solar radiation to the desired illuminance values. We have converted the solar radiant energy values from four Weather Bureau stations[j] and the results for two of these are shown graphically in Fig. 8. The amount of solar radiant energy reaching the earth's surface

[j] These stations are:

	N. Lat.	W. Long.	Altitude	Years Covered
Washington, D. C.	38°56′	77°05′	397 ft.	1914 to 1936 inc.
Madison, Wis.	43°05′	89°23′	974 ft.	1911 to 1936 inc.
Lincoln, Nebr.	40°13′	96°41′	1225 ft.	1915 to 1936 inc.
Blue Hill, Mass.	42°13′	71°02′	640 ft.	1933 to 1936 inc.

of course varies throughout the year. In Fig. 8 the maxima, minima, and average for Washington, D. C., and Lincoln, Nebraska, are shown. The values for the Wisconsin station agree quite closely with those of the Nebraska station, while those obtained at Blue Hill, Massachusetts, agree fairly well with the Washington values.

Curve *A* in Fig. 8 shows the values of the "illumination equivalents" as determined at Washington, D. C. The application of these "luminous equivalents" to "solar radiation" measurements made at other stations where the average water-vapor content differs from that of the average water-vapor content at Washington may be subject to some criticism. It should be possible, theoretically, to apply a correction provided the mean vapor pressure at the two stations in question is known. Kimball[18] points out (p. 771) that the theoretical water-vapor pressure correction does not agree very well with the observed facts and, commenting on this discrepancy, he says, "In this present case, the greater decrease in radiation with increase in vapor pressure must be attributed to the fact that generally there is increased haziness and, therefore, increased scattering of the sun's rays with increase in vapor pressure partly perhaps on account of the hygroscopic character of the dust and other particles in the atmosphere."

This discrepancy draws attention to the uncertainty of the meaning of the term "water vapor" as used by various writers. In some cases, the term "water vapor" includes not only water in the gaseous state, but precipitated or condensed water in the form of very small particles. It also emphasizes the fact that visual inspection of the atmospheric condition may not be a precise criterion of the presence of small quantities of condensed water vapor (i.e. water droplets). The term "clear day," therefore, is not a precise one and averages of measurements made on "clear days" probably represent a condition in which some *condensed* water vapor as well as water vapor is present.

Referring to Fig. 8, it will be seen that, for the averaged Washington measurements, the illumination on a horizontal plane with the sun at the zenith is 9750 foot-candles. This agrees well with the value computed from Moon's proposed standard solar irradiance data, which is 9570. It should be remembered that Moon's standard curve applies to a condition where there is 20 mm of perceptible water and 300 dust particles per cubic centimeter. This probably approaches closely the average atmospheric condition at the Washington station. If Moon's curve is corrected to represent the atmospheric condition: $p = 760$, water $= 0$, dust $= 0$, ozone $= 2.8$ mm., the I value for solar altitude 90° becomes 10,700 foot-candles.

Referring again to Fig. 8, it will be seen that for the Lincoln, Nebraska, station, the average illuminance on the horizontal plane for zenith sun goes up to 10,300, which is somewhat lower than the December value measured at Washington, but higher than the Washington average. Inspection of the Weather Bureau's records of water-vapor pressure[18] indicates that the amount (yearly average) of water vapor present in the atmosphere at Lincoln, Nebraska, and Madison, Wisconsin, is very appreciably lower than at Washington, D. C. We should, therefore, expect somewhat higher values of direct sunlight illuminance at these western stations. It is true that a decrease in the amount of water in the atmosphere should be accompanied by a more rapid increase of solar irradiance than of solar illuminance. It is possible, therefore, that the Nebraska and Wisconsin values as typified by the curves in Fig. 8 are slightly greater than they should be. This is supported by the fact that the maximum value for the Lincoln station, 11,600, is greater than that derived from Moon's data for perfectly dry and dust-free air which is only 10,700.

For our purpose, it is permissible to take the average of all the data from the four Weather Bureau stations (as shown in the previous footnote) as representative of the most probable evaluation of solar illuminance by the U. S. Weather Bureau. This average is shown as Curve *D* in Fig. 7.

These averaged Weather Bureau data are only about 5 percent higher than Moon's computed value. If we wish to give equal weight (which seems a doubtful procedure in view of the great volume of Weather Bureau observations) to each of the three groups of experimental data shown in Fig. 7, the average curve would fall just slightly below Moon's computed value. We have, therefore, decided to use Moon's computed values as the satisfactory and most probable representation, for all parts of the world, of the illuminance on the horizontal plane due to direct sunlight for the atmospheric condition which is characterized by the visual judgment as *clear*.

B. Skylight, Clear Atmosphere

The data on the amount of radiant energy and light coming to the earth from the sky are less voluminous than those on direct solar radiant energy and illuminance. Moreover, there seems to be greater disagreement between the measurements made by observers of illuminance due to skylight than of solar illuminance. One of the most extensive investigations of sky luminance and illuminance due to skylight is that made by Kimball and his co-workers of the U. S. Weather Bureau. A full year's program of sky luminance measurements was completed in April, 1922. During two months of this period, observations were made at Chicago, Illinois, and during the remaining ten months, in a suburb of the city of Washington, D. C., that is practically free from smoke. Because of large quantities of smoke and other atmospheric contamination, the Chicago data seem to be of little use in arriving at acceptable values for the *clear* atmospheric condition. We shall, therefore, use only the observations made at Washington, D. C. Kimball and Hand[41] state that, in all, about 55,000 photometric readings of sky luminance and 9000 photometric readings of illuminance were made during the work carried on in Washington. The measurements of sky luminance were made at 2, 15, 30, 45, 60, 75, and 90 degrees in altitude above the horizon and at 0, 45, 90, 135, and 180 degrees in azimuth from the sun. Sufficient accuracy for this work is obtained by averaging the summer and winter values. The results of their measurements as published are given in terms of ratios to zenith luminance for each solar altitude. These have been converted to actual sky luminance values and are shown in Table II.[k] In this table the summer and winter observations are shown separately although, as just stated, for our final evaluation we shall deal with the *average* of the two groups of data, which is shown in Table III.

TABLE II. Luminance of the clear sky, winter and summer values.

Solar altitude	Azimuth from sun	Altitude for luminance values 2°	15°	30°	45°	60°	75°	90°
		CLEAR SKY, WINTER						
0°	0°	319	242	109	60.5	36.3	29.0	25.2
	45	111	120	71.3	49.9	34.3	29.0	
	90	61.2	69.6	52.4	37.0	30.2	27.7	
	135	75.3	86.4	60.7	39.8	29.5	26.0	
	180	96.5	106	69.0	44.6	31.5	27.2	
20°	0°	6140	—	2550	1110	590	371	261
	45	2330	1450	981	671	457	331	
	90	1030	679	441	339	287	277	
	135	1020	611	371	264	227	230	
	180	1140	666	371	258	211	206	
40°	0°	4190	3450	4780	—	1380	774	506
	45	2310	1750	1400	1120	891	688	
	90	1330	840	567	506	450	481	
	135	1270	673	415	344	334	380	
	180	1400	729	410	304	293	349	
		CLEAR SKY, SUMMER						
20°	0°	7850	—	4040	1510	870	521	372
	45	2870	2190	1450	1020	722	487	
	90	1270	1050	662	502	398	424	
	135	923	692	458	309	268	305	
	180	1060	792	443	316	275	283	
40°	0°	5480	4620	6290	—	2050	1180	746
	45	3100	2330	2010	1830	1400	1060	
	90	1590	1050	843	724	739	776	
	135	1300	843	552	448	463	552	
	180	1370	828	507	403	395	507	
60°	0°	3180	2660	2820	3840	—	2520	1530
	45	2660	2340	1990	2340	2560	2160	
	90	1770	1350	1100	1130	1350	1510	
	135	1450	933	719	765	887	1150	
	180	1530	979	719	765	826	1100	
70°	0°	3100	2760	2420	3420	7060	—	2140
	45	2680	1930	1900	2290	2850	3270	
	90	1650	1390	1200	1160	1330	1900	
	135	1330	899	792	813	942	1410	
	180	1240	835	663	663	877	1310	

To illustrate the relation between the sky luminance, B_s, and the illuminance, I_s, on a plane of some definite orientation at the earth's surface, reference is made to Fig. 9. Point O is on the plane P, on which the value of illuminance is desired. The line OA is the normal to this plane at O. The lines OL_1, OL_2, OL_3, and OL_4 lie at the corners of a rectangular pyramid having its apex at O. This pyramid encloses a solid angle ω. The average luminance of the element of sky included in this solid angle ω is B_s. The angle α is that included between the normal to the surface P and the midpoint of the solid angular element ω. If ω is expressed in terms of steradians, the unit in which solid angle is usually evaluated, then the illuminance on the surface P due to the element of sky lying within the solid angle ω is given by

$$I_s = \omega \frac{B_s}{\pi} \cos \alpha. \tag{23}$$

For the special case of the horizontal plane, at a point on which the sky hemisphere subtends a

[k] We have also converted Kimball's values, which are given in *millilamberts*, to *foot-candles*, the luminance unit used throughout this paper.

TABLE III. Luminance of the clear sky, average of winter and summer values.

Solar altitude	Azimuth from sun	Altitude for luminance values 2°	15°	30°	45°	60°	75°	90°
0°	0°	364	294	141	71.4	44.8	34.8	30.5
	45	124	151	88.4	62.9	44.2	35.8	
	90	68.2	88.7	65.5	45.9	36.1	35.0	
	135	71.7	92.0	68.7	43.2	32.2	30.3	
	180	93.1	116	75.6	49.5	36.2	32.2	
20°	0°	7000	—	3300	1310	730	446	316
	45	2600	1820	1220	846	590	409	
	90	1150	864	552	420	342	350	
	135	972	652	414	286	248	268	
	180	1100	729	407	287	243	244	
40°	0°	4840	4040	5540	—	1720	977	626
	45	2700	2040	1700	1480	1150	874	
	90	1460	945	705	615	594	628	
	135	1280	758	484	396	398	466	
	180	1380	778	458	354	344	428	
60°	0°	2810	2320	2480	3290	—	2090	1280
	45	2320	2050	1690	1890	2090	1780	
	90	1620	1220	920	960	1090	1220	
	135	1430	839	630	676	763	971	
	180	1550	920	650	671	719	928	
70°	0°	2740	2410	2130	2930	5910	—	1790
	45	2340	1690	1610	1850	2330	2700	
	90	1510	1250	1000	985	1070	1540	
	135	1310	808	694	719	810	1190	
	180	1250	785	600	581	764	1110	

solid angle of 2π steradians, and for a uniformly bright sky, the illuminance on the horizontal plane, in foot-candles, is equal to the luminance of the sky, in foot-lamberts.

Unfortunately, when the atmosphere is *clear*, the sky is not uniformly bright. The maximum luminance is found in the immediate vicinity of the sun. This decreases, as the angle from the sun increases, to a minimum at 90 degrees. Hence, with the sun at the zenith, a symmetrical distribution of luminance exists around the zenith. This permits the formulation of the functional relation between B_s and the angular distance from the sun. For this condition, it is possible to use the well-known methods of integral calculus for the computation of the illuminance on a plane at the earth's surface. When the sun is not at the zenith, the luminance distribution is such that it becomes very difficult to apply the usual integration methods. Other methods, which perhaps lack the elegance of integral calculus but which give results of ample precision for all practical purposes, are usually used.

These methods involve the subdivision of the total solid angle subtended by the sky, at a point on the plane on which the illuminance is desired, into a large number of equal solid angles, ω_1, ω_2, $\cdots\omega_n$. If such elements are small enough so that: (a) the luminance of the element of sky subtended is approximately uniform, and (b) $\Delta\alpha$ (Fig. 9) is not large enough to cause serious error (as a result of the $\cos\alpha$ term in Eq. (23)), the illuminance on the chosen plane for each solid angular element may be computed by the use of Eq. (23). The sum of all the illuminances thus found gives the desired value and is shown formally as follows:

$$I_s=\omega_1\cdot(B_{s1}/\pi)\cos\alpha_1+\omega_2(B_{s2}/\pi)\cos\alpha_2 + \cdots\omega_n(B_{sn}/\pi)\cos\alpha_n. \quad (24)$$

By applying this method to Kimball's data, the illuminance on the horizontal plane for values of solar altitude was calculated. These results, computed from the average of the winter and summer Kimball measurements, are shown by Curve *A* in Fig. 10. Kimball observed higher luminance values and a less pure blue color during the summer months than during the winter months. Curves *B* and *C* in Fig. 11 show

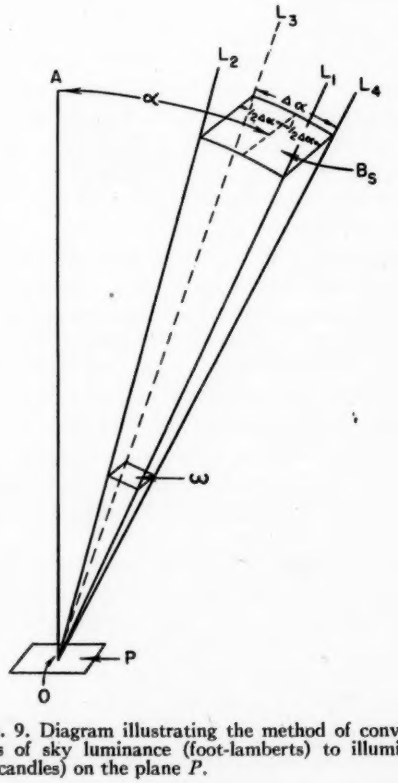


FIG. 9. Diagram illustrating the method of converting values of sky luminance (foot-lamberts) to illuminance (foot-candles) on the plane *P*.

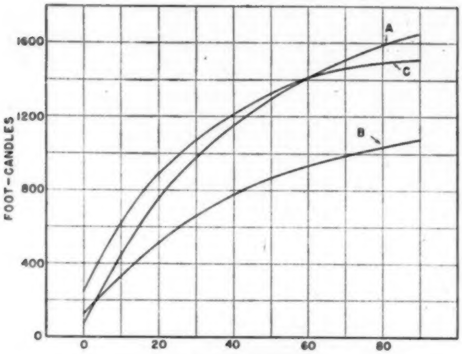


FIG. 10. Illuminance (foot-candles) on the horizontal plane due to light from the clear sky as a function of solar altitude.

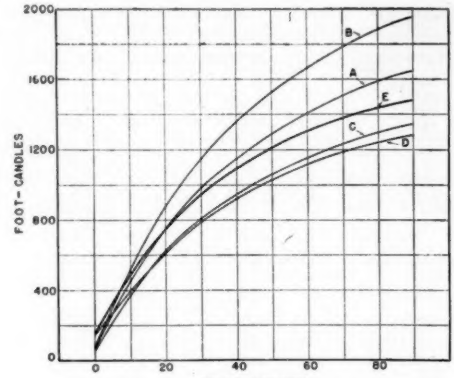


FIG. 11. Illuminance (foot-candles) on the horizontal plane due to light from the clear sky as a function of solar altitude.

this variation. These results may be compared with Curve *A* in Fig. 11, which is the average for winter and summer and is identical with *A* in Fig. 10. It will be seen that the summer and winter values deviate about ±18 percent from the year's average. The sky illuminance measurements reported by Elvegård and Sjöstedt[37] are shown as Curve *C* in Fig. 10. It should be remembered that all values above a solar altitude of 50 degrees were obtained by extrapolation. Their measurements agree fairly well with the Kimball observations. Since the original publication containing these measurements could not be obtained, we have no definite information as to the clarity of the skies when the measurements were made, except for the qualitative statement that they do refer to clear sky conditions. In localities near large bodies of water, the water vapor content of the atmosphere tends to be high, and it seems likely therefore that Curve *C* represents an average which includes some slightly hazy skies due to the presence of some condensed water vapor. In fact, it probably represents a condition similar to the average to which the Washington observations pertain.

The values of illuminance on the horizontal plane due to skylight obtained by Kunerth and Miller[36] are shown as Curve *B* in Fig. 10. These are very low compared with the other two groups of data. The sunlight illuminance values obtained by Kunerth and Miller have been shown previously (Curve *B*, Fig. 7). They also are lower than those reported by other observers. The fact that both sunlight and skylight values measured at the Iowa station are low compared with the other observations cannot be explained by assuming that the particular atmospheric conditions under which they were obtained were different. Had the sunlight values been high, it might be reasonably assumed that the amount of condensed water vapor present in the atmosphere when the sky appeared to be "clear" as judged by visual observations was appreciably less at the Iowa station than at the Washington and the Scandinavian stations, thus resulting in skies of lower luminance and, hence, lower illuminance on the horizontal plane. It is probable that this condition actually did exist. The Iowa observers selected the days on which to make their measurements very carefully and used only 33 days during 19 months. Ames, Iowa, is situated on the central continental plain far removed from any large body of water. The prevailing air movement is from west to east, and the air over this locality frequently contains very little water either in the condensed form or as vapor. Years of personal observation have convinced us that the atmospheric condition on the central plains which, by visual observation, is characterized as *clear* refers to a sky which has a more saturated blue color and is less bright than one on the eastern seaboard, which, as a result of visual observation, is described by the same term, *clear*. Such an atmospheric condition should result in higher sunlight values in the plains area. The Weather

Bureau measurements (Fig. 8) actually show a higher average for sunlight illuminance at the Nebraska and Wisconsin stations.

Since *both* the sunlight and skylight values are low, we can only conclude that, for some unknown reason, the measurements obtained at the Iowa station are lower than they should be. By increasing the sunlight values 20 percent, the Iowa data can be made to conform very closely to the average of the sunlight illuminance computed from the Weather Bureau observations made at Lincoln, Nebraska. By increasing the Iowa skylight illuminance values 20 percent, Curve *D* in Fig. 11 is obtained. It will be noted that this agrees quite well with Curve *C* in Fig. 11 for the average winter data obtained at Washington, D. C. The difference between the summer and winter sky luminance measurements made at Washington, D. C., can only be ascribed to less scattering of sunlight because of the smaller amount of condensed water vapor (or possibly less dust) present in the atmosphere during the winter months. If the scattering of sunlight is the same during winter and summer, the winter sky luminance should be higher than the summer because during this season the solar distance is about seven percent less than during winter.

For the final evaluation of illumination due to skylight, three groups of measurements have been averaged. They are:

1. The Scandinavian data as published by Elvegård and Sjöstedt (Curve *C*, Fig. 10).
2. The average of all measurements, winter and summer, made by Kimball at Washington, D. C. (Curve *A*, Fig. 11).
3. The values obtained by Kunerth and Miller at Ames, Iowa, to which have been applied an assumed correction of +20 percent (Curve *D*, Fig. 11).

The result of this averaging yields Curve *E* in Fig. 11.

As a final result of this analysis of the available data on skylight and sunlight intensities two groups of data have been obtained, one giving the relationship between the illuminance due to direct sunlight on the horizontal plane and solar altitude (Curve *A*, Fig. 7), and the other giving the same functional relationship for illuminance due to skylight (Curve *E*, Fig. 11).

It is considered that these values apply to an atmospheric condition which, as judged by visual inspection, may be characterized by the verbal description *clear*. It is unlikely, however, that they do apply precisely to an atmosphere which actually contains no condensed water vapor or dust. It does not seem probable that the departure of the values as embodied in the two curves mentioned above from this condition of *perfect clarity* is of sufficient magnitude to be of any appreciable importance for the purpose of predicting correct camera exposures.

The magnitude of the departure of the values which we propose to use from perfect atmospheric clarity may be evaluated by using some observations made on Mount Whitney, California.[31] The elevation of this station is 14,170 feet, barometric pressure 440 mm, ozone 1.8 mm. At this elevation there is no dust and practically no condensed water vapor in the atmosphere above the station. By computing from the spectroradiometric measurements, the luminous transmittance of the atmosphere above Mount Whitney is found to be 0.93. This corrected for sea level ($m=1$) gives a transmission of 0.90. Using Moon's extrapolated value for the illuminance on the normal plane just outside of the earth's atmosphere, 12,000 foot-candles, it is found that for sun at the zenith with the entire atmosphere of the same composition as that above Mount Whitney, the illuminance due to direct sunlight on the horizontal plane at sea level should be 10,800 foot-candles. This value may also be checked by using Moon's proposed standard solar radiation curve as shown in Fig. 7, which applies to an atmosphere containing 20 mm of precipitable water, 300 dust particles per cubic centimeter, and 2.8 mm of ozone. Computation for zero water and zero dust gives a value of 10,700 foot-candles on the horizontal plane at sea level with sun at the zenith. If 10,750 be taken as the most probable value for a water-free and dust-free atmosphere, then the corresponding value derived from Curve *A*, Fig. 7, is only 11 percent below this ideal maximum.

Since less sunlight is scattered by a water-free, dust-free atmosphere, the illuminance due to skylight should probably be somewhat lower than that shown in Curve *E*, Fig. 11. It is difficult to estimate just how much lower. These minor uncertainties can only be resolved by further measurements made under more precisely defined atmospheric conditions. However, we feel confident that the values proposed in this paper are of ample precision for the purpose in hand.

IV. THE EVALUATION OF THE AVAILABLE SUNLIGHT AND SKYLIGHT FOR PHOTOGRAPHIC PURPOSES

In the previous section the amount of light reaching the earth's surface was expressed in terms of the illuminance on a horizontal plane. The question now arises, is the illuminance on this plane the best criterion of the camera exposure, t/f^2, required to yield a correctly exposed negative? Before answering this question, other possible methods of evaluating the "illuminating power" of solar radiant energy should be considered.

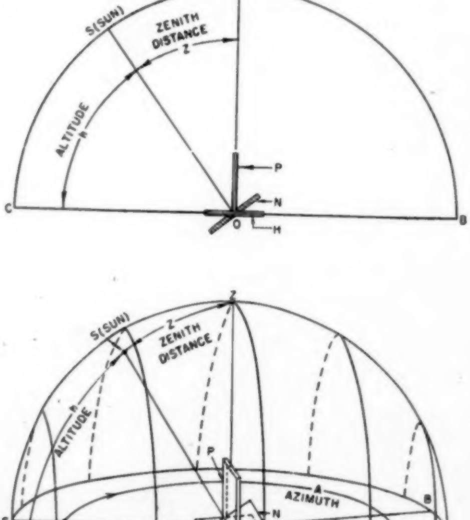


FIG. 12. Diagram illustrating the relationship between three possible methods of evaluating the illuminance attributable to sunlight and skylight. These include: (a) the illuminance on the normal (N) plane; (b) the illuminance on the horizontal (H) plane; and (c) the illuminance on the perpendicular (P) plane.

A. Illuminance on the Normal, Perpendicular,[1] and Horizontal Planes

Among the many ways of evaluating this illuminating power there are at least two, in addition to the use of the horizontal plane, which deserve attention. These have been used extensively by illuminating engineers, meteorologists, and others interested in illumination coming from natural sources. Presumably they have been found significant and useful in these fields and they may be of equal significance for the calculation of required camera exposures.

These methods are the illuminance on a *normal plane* and that on a *perpendicular plane facing the sun*. The term "facing the sun" means that the azimuth of a normal to that plane is the same as that of the sun. The relationship between these two planes and the horizontal plane is illustrated in Fig. 12. The upper part of this figure represents a perpendicular plane through the line OS which is drawn from a point O on the earth's surface to the sun, S, and intersecting the horizontal plane along the line CB. In the upper part of Fig. 12, H represents a cross section through an element of the horizontal plane; N, a cross section through an element of the normal plane; and P, a cross section through an element of the perpendicular plane facing the sun. The lower part of Fig. 12 illustrates the three-dimensional relationships and, together with the cross-sectional view above, serves to define the terms *solar azimuth*, A, *solar altitude*, h, *and solar zenith distance*, z, all of which are expressed in angular units.

Since we must deal with several aspects of solar illumination and with different methods of evaluating these aspects, a systematic nomenclature is needed to avoid confusion. The symbol I will be used to designate illuminance. The subscripts n, p, and h will be used to designate the evaluation of illumination on the *normal*, *perpendicular* (facing the sun), and *horizontal* planes. The additional subscripts d, s, and t will be used to designate *direct sunlight*, *skylight*, and *total*

[1] In this communication the word *perpendicular*, when used to indicate a plane upon which direct sunlight and skylight are evaluated, is defined as that plane which is *perpendicular to the horizontal plane*. It will be denoted by the symbol P and in some cases by the subscript letter p. The preferable term *vertical* is not used because we wish to use the symbol V and the subscript v in referring to the volume density of luminous energy, that is, luminous density.

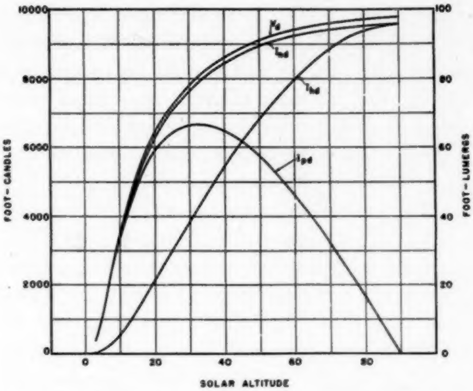


FIG. 13. Illuminance (foot-candles) due to sunlight (on the normal, horizontal, and perpendicular planes) as a function of solar altitude; and luminous density (foot-lumergs) due to sunlight as a function of solar altitude.

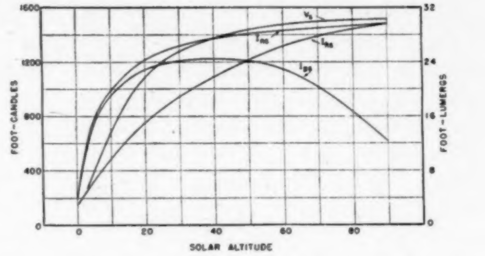


FIG. 14. Illuminance (foot-candles) due to skylight (on the normal, horizontal, and perpendicular planes) as a function of solar altitude; and luminous density f (foot-lumergs) due to skylight as a function of solar altitude.

light (direct sunlight plus skylight). Thus, I_{pd} indicates the illumination on a perpendicular plane (facing the sun) which is due to direct sunlight.

By using the data already presented, those contained in Curve *A* of Fig. 7, Curve *E*, Fig. 11, and in Table III, the illuminance on the *n* and *p* planes can be computed. In the case of direct sunlight, this is a fairly simple operation, involving only the use of the cosine of the angle between the respective planes. In the case of skylight, a method similar to that explained previously must be employed. The portion of the sky hemisphere which is visible from a point on the plane in question must be subdivided into a large number of equal solid angular elements, the contribution to the illuminance on the plane by each element determined by (23), and the sum of all of these computed. This operation, while not difficult, is long and laborious.

The illuminance as a function of solar altitude on the normal, perpendicular, and horizontal planes is shown by the curves, which are designated by symbols in accord with the nomenclature of the previous paragraph, in Figs. 13, 14, and 15. Figure 13 contains the information relative to direct sunlight; Fig. 14, that pertaining to skylight; and Fig. 15, the total illuminance values.

Since some readers may wish to use the numerical values, represented by the curves in Figs. 13, 14, and 15, for their own computations, those for the illuminance on the horizontal and perpendicular planes are given in Table IV. Those for the normal plane appear in Table VII.

B. Correct Camera Exposure

Before attempting to appraise the merits and shortcomings of these methods of evaluating the available light, it is necessary to define clearly the purpose for which such evaluation is to be used, i.e., the determination of the *correct camera exposure*. The most rational definition of this term is that the *correct* camera exposure is the one which is necessary and sufficient for the production of a negative from which an *excellent* print can be made. Any value of camera exposure less than this is insufficient and leads at once to

TABLE IV. Illuminance due to direct sunlight, skylight, and total on horizontal and vertical planes.

Solar altitude h	Air mass m	Direct sunlight I_{hd} (ft.-c)	Direct sunlight I_{pd} (ft.-c)	Skylight I_{hs} (ft.-c)	Skylight I_{ps} (ft.-c)	Total I_{ht} (ft.-c)	Total I_{pt} (ft.-c)
3	15.36	19.6	374	256	587	277	961
5	10.39	100	1150	325	746	425	1900
7	7.77	252	2050	395	848	647	2900
10	5.60	590	3350	491	953	1080	4300
15	3.82	1310	3910	629	1070	1940	5980
20	2.90	2130	5860	750	1140	2880	7000
25	2.36	2980	6390	856	1180	3840	7570
30	2.00	3820	6620	945	1210	4760	7830
35	1.74	4650	6640	1020	1220	5670	7860
40	1.55	5440	6490	1090	1220	6530	7710
45	1.41	6170	6170	1160	1220	7330	7390
50	1.30	6850	5750	1210	1200	8060	6950
55	1.22	7450	5220	1270	1180	8720	6400
60	1.15	8000	4620	1310	1150	9310	5770
65	1.10	8470	3950	1350	1090	9820	5040
70	1.06	8860	3230	1390	1020	10250	4250
75	1.04	9160	2450	1420	930	10580	3380
80	1.02	9380	1650	1440	834	10820	2480
85	1.01	9510	833	1460	728	10970	1560
90	1.00	9570	0	1480	615	11050	615

loss of print quality. Any value greater than this is unnecessary. In general, the correct camera exposure can be exceeded by relatively large amounts without incurring serious loss of print quality. However, such additional camera exposure is not necessary and may lead to undesirable results. The correct camera exposure is therefore the *least* which will yield a negative from which an excellent print can be made.[m]

C. Aspects of Object Luminance as Criteria of Correct Camera Exposure

In the use of instruments to determine the required camera exposure, some particular aspect of object luminance, B_o, must be selected for

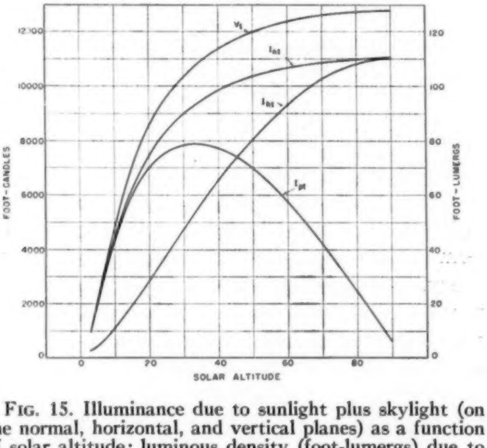


FIG. 15. Illuminance due to sunlight plus skylight (on the normal, horizontal, and vertical planes) as a function of solar altitude; luminous density (foot-lumergs) due to sunlight plus skylight as a function of solar altitude.

[m] The desirability of defining *correct camera exposure* in this manner may be questioned. We admit that in practice it is undesirable to aim at giving the correct exposure as defined above. If every variable upon which camera exposure depends were precisely known, there would be little objection to aiming at correct camera exposure as defined. In practice, however, some of the variables are not known precisely. They may be measured with a certain amount of experimental error or estimated, subject to some uncertainty, by visual inspection of conditions. It is customary, therefore, in practice, to aim at a camera exposure somewhat greater than that which is defined above as *correct*. This desirable end is accomplished by inserting into the computations a safety factor. The magnitude of the safety factor cannot be the same for all photographic materials because of variations in exposure latitude. Thus, for black-and-white negative-positive processes, it is permissible and desirable to use a larger safety factor than can be tolerated in the case of reversal or color processes. In view of these facts, we prefer to define correct camera exposure as the least which will give a negative from which an excellent print can be made and to assure the giving of ample camera exposure by the use of a safety factor chosen to suit the process being used.

computations. Such an aspect may be referred to as a *criterion* of camera exposure. This criterion must be measurable and should be unique and meaningful. The most obvious aspects of scene luminance which meet the requirements of measurability and uniqueness are the *minimum* luminance, B_o min; the *maximum* luminance, B_o max; and the integrated luminance, $B_o(M)$.[42, n] It should be remembered that values of scene luminance, as measured by photoelectric exposure meters of the integrating type, are not, in general, values of *average* scene luminance, but they are more properly designated as values of *integrated* scene luminance, $B_o(M)$. They possess only limited uniqueness. Scenes differing with respect to luminance *distribution* may give the same value of average luminance but have widely different values of integrated luminance. Nevertheless, it has been found that instruments of this type give values which, because the luminance distribution characteristics of most scenes do not depart too widely from the average mode of luminance distribution, are useful in indicating the camera exposure which will yield a satisfactory negative. However, such an integrated value of scene luminance cannot be regarded as a *unique* characteristic of the scene except in the sense that it may be unique for a particular exposure meter having a definite "acceptance angle," and a definite "angular response function."

Long before the application of scientific mensurational methods to photography, the precept "expose for the shadows and the highlights will take care of themselves" had become almost axiomatic. Our most recent knowledge of the theory and practice of tone reproduction supports the soundness of this rule. As originally formulated, this rule applied to the black-and-white, negative-positive photographic process and in the light of our present knowledge, it appears to be valid for any negative-positive process yielding either black-and-white or color results. It does not, however, apply to reversal processes.

1. Minimum object luminance.—The experiential soundness of the "shadow exposure rule" suggests the probable excellence of B_o min as the

[n] See reference 42, footnote p. 559,

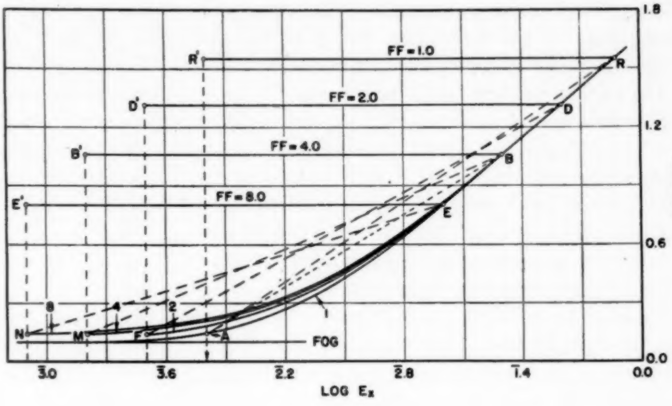


FIG. 16. Curves showing the consequences of using the actual values of flare factor (*FF*) for various scenes (if known) in the computation of camera exposure, *CE*, by use of Eq. (20a).

most significant criterion of correct camera exposure. Obviously, no criterion, regardless of its excellence, can give the desired result unless it is properly used. How can this be done? It might be expected that it should only be necessary to employ the camera exposure equation,

$$t/f^2 = 1/B_o \min \cdot FF \cdot K \cdot S. \qquad \text{(20a)}$$

This cannot be done since the value of flare factor for a particular scene is, in general, unknown and cannot be determined conveniently in the field. The flare factor, even for the same camera-lens equipment, varies from scene to scene over a wide range, from at least 1.25 to 9.0. Moreover, even if the flare factor were known, the use of (20a) would result, for all scenes, in the rendition of the scene element of minimum luminance at a fixed point on the D_x-$\log E_x$ curve. To realize that this is undesirable, one need only analyze the consequences of such procedure.

In Fig. 16 is shown one of the cases used in the establishment of the fractional gradient speed criterion. Curve 1 is the D_x-$\log E_x$ characteristic for the ***negative material***. The negative from which the first excellent print was made (as determined by the average judgment of 200 observers) used the part of the characteristic curve between the points *A* and *B*. The average gradient, $\bar{G}_x$, of this portion is 0.61, the slope of the straight line, *AB*. The gradient at point *A*, at which the scene element of minimum luminance was rendered, is 0.20 (rounded off from 0.195). This minimum limiting gradient, G_x min, divided by the average gradient, gives the *shadow detail compression factor* (more briefly termed the *fractional gradient*), $K_x = 0.33$.

The scene chosen (Willow Pond) for use in the establishment of the fractional gradient criterion of effective camera speed, when photographed with the particular camera equipment used for that purpose, gave an illuminance scale, IS_i, on the negative material of 28.[11] This value was rounded off to 32, thus making log of $IS_i = 1.50$. In a subsequent investigation[10] of the distribution of luminance in exterior scenes, it was found that the average luminance scale, BS_0, for 126 exterior scenes of various types was 160, $\log BS_o = 2.2$. With the camera lens equipment used in photographing these scenes, the average value of flare factor was found to be 2.35. From a study of the flare characteristics of a large number of different camera-lens combinations used very extensively in the amateur field, it was concluded that the average flare factor applicable to this field of photography is probably about 4.0 rather than 2.35. The average luminance scale, BS_o, assuming an average value of $FF = 4.0$, leads to an average illuminance scale on the negative material of 40, $\log BS_o = 1.6$. It is evident, therefore, that for the conditions which existed when the Willow Pond scene was photographed with the camera-lens equipment used, the negative material was called upon to render an illuminance scale very nearly equivalent in magnitude to the average illuminance scale which the negative materials in amateur practice are called upon to render.

The original luminance measurements on the

Willow Pond scene were made with a portable photometer which later was found to be unreliable because of excessive scattered light within the optical system. As a result, the value of minimum object luminance, B_o min, was subject to a large error. Subsequent measurements made with a portable photometer from which stray light had been almost completely eliminated indicate that the most probable value of B_o min should be approximately 20 foot-lamberts. Other corroborating evidence, namely, a study of the luminance scale, BS_o, and B_o min values for other scenes which obviously resemble the Willow Pond scene very closely with respect to brightness distribution, indicate that the luminance scale of the Willow Pond scene was approximately 128, $\log IS_i = 2.10$. This is approximately equal to the average luminance scale, $\log IS_i = 2.20$, of the 126 scenes measured and reported in one of our communications.[10] This leads to a flare factor of 4.0 for the Willow Pond scene as photographed with the particular camera-lens equipment which was used. This value is the same as that which we have concluded, from a considerable volume of data, is applicable on the average in the field of amateur photography.

By using a value of 4 for flare factor, curve 4 (Fig. 16), from the point M to B, was constructed according to the technique previously discussed. This is the characteristic curve of the *negative*, $D_n = f(\log B_o)$. It is this curve, not curve 1, which determines the relationship between a given luminance difference, ΔB_o, in the object (scene) and the corresponding luminance difference, ΔB_r, in the positive, the reproduction. This curve is the *direct determinant* of tone reproduction quality. The gradient, G_m, at the point M is obtained by dividing the gradient at point A by the flare factor. In this case, $G_m = 0.20/4.0 = 0.05$. The average gradient of the negative, $\bar{G}_n$, is 0.44, the slope of the straight line from M to B. This

TABLE V. Data (from Fig. 16) illustrating the consequences of introducing a variable flare factor into Eq. (20a).

	Negative material			Negative			
FF	$\bar{G}_z$	G_z min	K_z	DS_n	$\bar{G}_n$	G_n min	K_n
1	0.68	0.20	0.29	1.42	0.68	0.20	0.29
2	0.66	0.20	0.30	1.18	0.55	0.10	0.18
4	0.61	0.20	0.33	0.92	0.44	0.05	0.11
8	0.56	0.20	0.36	0.66	0.31	0.025	0.08

gives the ***shadow detail compression factor***, K_n, of the ***negative***. K_n must be very clearly distinguished from K_z, which is the shadow detail compression factor derived from that part of the *negative material* characteristic, $D_z = f(\log E_z)$, which is actually utilized in the rendition of an object. For the negative characteristic represented by curve 4 (Fig. 16),

$$K_n = G_m/\bar{G}_n$$
$$K_n = 0.05/0.44 = 0.11.$$

This is the *fundamental criterion* which determines the least camera exposure which will yield a negative from which an excellent print can be made. Any *negative* having a D_n-$\log B_o$ characteristic such that K_n is less than 0.11 will have received *insufficient* camera exposure to permit the making of an excellent print, while any negative having a characteristic for which K_n is greater than 0.11 will have received *more* exposure than is necessary for the making of an excellent print.[o]

By considering three other hypothetical scenes all having the same luminance scale, that is 128, as the Willow Pond scene, but having flare factors of 1, 2, and 8, respectively, we can examine the consequences of using the variable values of flare factor 1, 2, and 8 in the camera exposure formula (20a). The results are illustrated graphically in Fig. 16. The minimum scene brightness, B_o min, will in all cases be rendered at the point A on the negative material characteristic, the used positions being for the FF of 1.0 from A to R; for FF equal to 2.0 from A to D; for FF equal to 4.0 from A to B; and for FF

TABLE VI. Data (from Fig. 17) illustrating the consequences of using a fixed flare factor (FF=4.0) when photographing scenes in which the flare factor is variable.

	Negative material			Negative				
FF	$\bar{G}_z$	G_z min	K_z	DS_n	$\bar{G}_n$	G_n min	K_n	K_n'
1	0.46	0.00	0.00	0.96	0.46	0.00	0.00	0.15
2	0.53	0.06	0.11	0.95	0.45	0.03	0.07	0.17
4	0.61	0.20	0.33	0.92	0.44	0.05	0.11	0.18
8	0.67	0.38	0.57	0.83	0.40	0.05	0.12	0.20

[o] This statement is strictly valid only for negatives made from scenes all of which have an illuminance scale equal to 2.10. However, some evidence is at hand which indicates that the departures, in the case of scenes having lesser and greater luminance scales, from the specification of the adequacy of the $K_n = 0.11$ criterion, are of negligible magnitude.

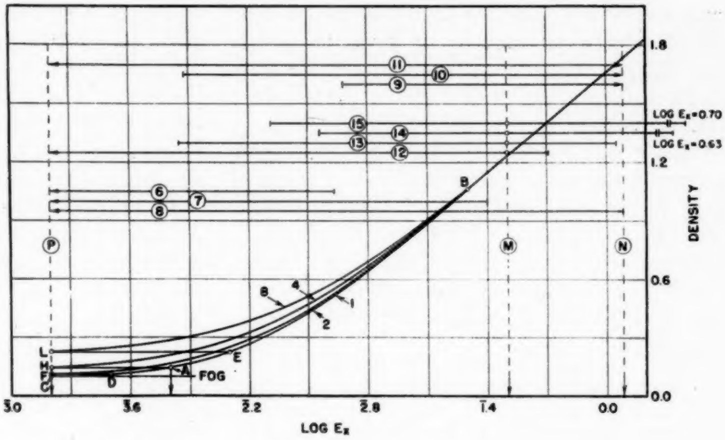


FIG. 17. Curves showing the consequences of using a constant value of flare factor ($FF=2.0$) in the computation of camera exposure, *CE*, by use of Eq. (20a).

equal to 8.0 from *A* to *E*. By using these same values of flare factor, the respective ***negative characteristics*** have been constructed and are designated in Fig. 16 as curves 1, 2, 4, and 8, respectively. For a flare factor of 1, the negative characteristic is identical to the used part of the negative material characteristic curve, *A* to *R*. For *FF* equal to 2, the negative characteristic extends from *F* to *D*; for *FF* equal to 4, from *M* to *B*; and for *FF* equal to 8, from *N* to *E*. It will be seen that the use of a variable flare factor in the camera exposure formula (20a) leads to negatives having the same value of minimum density, D_n min. The value of maximum negative density however decreases from 1.55 when a flare factor of 1.0 was used, and to 0.8 when a flare factor of 8.0 was used. In Table V are shown some of the significant numerical values which are characteristic of the used portions of the ***negative material characteristic*** and of the resultant ***negatives***. It will be noted that the value of K_x, the shadow detail compression factor for the ***negative material***, increases when increasing values of flare factor are used in the camera exposure formula (20a). This is to be expected since G_x min must necessarily be constant, while the value of $\bar{G}_x$ must decrease because the density scale of the negative decreases.

Of much greater significance, however, in deciding upon the correct camera exposure are the corresponding characteristics of the ***negative curves*** shown in the second part of this table. It will be seen that the values of K_n decrease steadily with increasing values of flare factor. From the results of extensive psychophysical studies we know that the camera exposure used in connection with the flare factor of 4 yielded a negative which was correctly exposed in the sense of our definition, namely, that less exposure was insufficient to yield a print of excellent quality and that more exposure was unnecessary, since it resulted in little if any increase in print quality. For this negative curve (*M* to *B*), $K_n=0.11$. Hence, using a flare factor of less than 4 in formula (20a) leads to more camera exposure than is necessary since values of K_n for $FF=1$ and $FF=2$ are 0.29 and 0.18, respectively. On the other hand, the use of a flare factor greater than 4, as shown in the last line of Table V, leads to a value of K_n which is somewhat less than is required to yield a negative from which an excellent print can be made. Moreover, this technique leads to negatives, of scenes having the same value of illuminance scale, which have widely different values of density scale, DS_n, thus necessitating the use of several grades of printing papers differing markedly with respect to available exposure scale. We conclude from this analysis, therefore, that even if the flare factor for each individual scene were known or could be measured conveniently, the use of that value in the camera exposure formula (20a) would not be entirely satisfactory.

The best alternative, therefore, seems to be to consider the consequences of using some constant value of flare factor in Eq. (20a). In choosing such a constant value, we shall take the position that it should be such as to yield, for scenes and camera equipment having average flare characteristics, negatives which have received *just sufficient* exposure so that excellent prints can be made therefrom. Since we have previously expressed the opinion that the average value of flare factor met within the amateur field is probably approximately 4.0, we shall examine the consequences of using that number in Eq. (20a).

In Fig. 17 are shown graphically the results of this procedure. The expression for camera exposure now becomes

$$t/f^2 = 1/4B_o \min \cdot K \cdot S. \qquad (20b)$$

The curve marked 1 is the D_x^*-$\log E_x$ characteristic of the negative material and is identical to curve 1 in Fig. 16. The used portions of this characteristic curve for scenes having average luminance scales and flare factors of 1, 2, 4, and 8 are represented by the curves C to B, D to B, A to B, and E to B, respectively. Values of minimum gradient, G_x min, as shown in Table VI, vary from 0.00 to 0.38.

The negative characteristic curves,

$$D_n = f(\log B_o),$$

have been computed for flare factors of 1, 2, 4, and 8, and are shown in Fig. 17 as curve No. 1 extending from C to B; curve No. 2 extending from F to B; curve No. 4 extending from H to B; and curve No. 8 extending from L to B. In this case the simplified construction which makes the high density end of the curves coincident has been used. This introduces a very small error since the maximum displacement in $\log E$ is only 0.03. The pertinent numerical characteristics of the curves shown in Fig. 17 are given in Table VI. It will be noted that the values of K_x now increase rather rapidly. This results from the rendering of the scene element of minimum luminance at constantly increasing densities on the characteristic curve of the negative material. Values of K_x less than 0.33 or greater than 0.33 do not, however, necessarily indicate insufficient or excessive exposure. Reference must be made to values of K_n in the second part of the table to decide on this condition. When the scene has a flare factor of 1, the fractional gradient, K_n, has become zero, and we know that any value of K_n less than 0.11 indicates that an excellent print cannot be made from this negative. When the flare factor is 2, the value of K_n is still somewhat lower than is necessary for the production of an excellent print, while, for a scene in which the flare factor is 8, K_n is slightly greater than that required for the making of an excellent print.

It is apparent, therefore, that the use of a constant value of $FF = 4.0$, while giving just sufficient camera exposure to result in excellent print quality for the *average* condition which exists in the amateur field, will lead to insufficient exposure in the case of scenes where the actual flare factor is less than 4.0. We shall adopt the position, however, that the use of 4 in Eq. (20a) is satisfactory and shall incorporate in the final camera exposure formula a partial safety factor, SF', of suitable magnitude to eliminate the possibility of obtaining underexposed negatives when flare factors less than 4.0 are encountered in scenes having luminance scales of the average magnitude. A tentative value of 2 for such a safety factor is suggested.

It is of interest to examine the effect of using such a safety factor upon the values of K_n. This is shown in the last column of Table VI under the caption K_n'. It will be seen that this safety factor is of sufficient magnitude so that all values of K_n' are now considerably above the minimum, namely 0.11, which is required for the production of prints of excellent quality.

We shall now consider the way in which scenes varying widely in luminance scale will be rendered by the application of the camera exposure formula (20b), using a constant value of $FF = 4.0$. For this purpose, previously published data[10] relative to the luminance distribution in exterior scenes will be used. The minimum luminance scale found was 27, $\log BS_o = 1.43$. This will be rendered as shown by the horizontal line 6 (Fig. 17). The maximum luminance scale observed was 750, $\log BS_o = 2.88$. This scene will be rendered as indicated by the line 8. The *average* luminance scale was found to be 160, $\log BS_o = 2.20$, and such a scene will be rendered as shown by line 7. Thus, this method of using B_o min results in D_n-$\log B_o$ curves, the low-luminance ends of which *all* lie at $\log E = \log E_{0.3\bar{G}-0.6}$. In other words, the exposure (mcs) due to *image-forming light*, E_{io}, is $I_{io} \cdot t$, incident on the negative material in the camera at the point where the scene element of minimum luminance is imaged, is constant and independent of flare factor and object luminance scale. This does not lead to a constant value of minimum negative density. For the average scene (line 7), the average flare factor to be expected with average amateur equipment is 4.0 which gives an average D_n min of 0.04 above fog. For the *average distant* scene having a luminance scale of 81 and a flare factor of approximately 3.5, the average D_n min is 0.03 above fog. For the average ***nearby sunlit scene*** with the object of chief interest in ***light shade***, the average luminance scale is 345 and the flare factor is 4.3. This leads to an average D_n min of 0.05 above fog. The point on the negative material at which the scene element of maximum luminance is imaged receives an exposure given by $\log E_{0.3\bar{G}}+(\log BS_o-0.6)$. For the scene of greatest luminance scale thus far found in practice, $\log BS_o$ is 2.88, giving a maximum exposure of $\log E_{0.3\bar{G}}+2.28$. The useful exposure scale, $\log ES_x$, of modern negative materials, as measured from the $\log E_{0.3\bar{G}}$ point is, in practically all cases, greater than 2.6. Hence, camera exposure values determined in this manner do not result in loss of highlight detail due to overexposure. The flare factor, in a few cases, may be as high as 8.0 or 10.0, but this occurs infrequently in ordinary terrestrial photography. A flare factor of 10.0 with the negative material illustrated in Fig. 17 gives D_n min equal to 0.17 plus fog. This may possibly be taken as an indication of too much camera exposure, but this is not the case. A lesser camera exposure would result in a value of K_n less than 0.11 which we know is required for the production of a print of excellent quality.

2. Maximum object luminance.—Since B_o max is one of the unique and measurable aspects of scene luminance, it should be considered as a possible criterion of correct camera exposure. The consequences of using this criterion are illustrated by the three horizontal lines near the top of Fig. 17. Line 11 represents the luminance scale of the scene of maximum probable luminance contrast, $\log BS_o=2.88$. The minimum luminance of this scene cannot be rendered to the left of the line P without loss of print quality. In the case of scenes having very high luminance scales associated with relatively low flare factors, it is possible that placing the scene element of minimum luminance on the line, P, may result in a negative characteristic having a value of K_n too low for the production of a print of excellent quality. The existing data concerning the rendition of scenes of this type are insufficient to warrant a positive conclusion. There is some indication, however, that under these conditions K_n, as defined previously, may fall below the value 0.11 without resulting in loss of print quality. The magnitude of the partial safety factor, SF', is sufficient, however, to cover all possible variations in flare factor even when these are associated with scenes of high luminance scale. If it were found necessary to render the scene element of minimum luminance at some point to the *right* of the line, P, the objections to the use of B_o max as a criterion of correct camera exposure would be considerably increased. Assuming that B_o min must fall on the line, P, it follows that B_o max must fall at a $\log E_x$ value given by $\log E_x=\log E_{0.3\bar{G}}+(\log BS_o-0.6)$. The vertical line N is located at this $\log E_x$ value. Now, the maximum B_o of *all* other scenes must be rendered by this same $\log E_x$ value. Hence, using a given negative material, the use of B_o max leads to negatives of constant maximum density, which, for the negative material shown in Fig. 17, is 1.72. Formula (20b) may now be modified for use with B_o max,

$$t/f^2=\max BS_o/4B_o \max\cdot K\cdot S. \qquad (20c)$$

The line 9 shows how the scene of minimum luminance scale will be rendered, and line 10 represents the average scene. D_n min for this group of scenes varies from a very low to a relatively high value, that for the average scene being 0.32 above fog. The camera exposure is correct *only* for the scene of maximum luminance scale, all others receiving ***more*** than is necessary. The average scene, by this method, receives 4.7 times as much camera exposure as is necessary, and the scene of minimum BS_o, 30 times. These conditions are undesirable and should be avoided, especially when this can be done so easily by using B_o min.

3. Integrated scene luminance.—To use this aspect of scene luminance as a criterion of camera exposure, it is necessary to refer again to the previously published data[10] which show the relation of $B_o(M)$ to B_o max and B_o min for a large number of scenes. Two new terms are needed now, namely, the ratio of $B_o(M)$ to B_o min and also B_o max to $B_o(M)$:

$$R_s = \frac{B_o(M)}{B_o \text{ min}},$$

$$R_h = \frac{B_o \text{ max}}{B_o(M)}.$$

The scene in which R_s is maximum will determine the least exposure by which the scene element having a luminance equal to the $B_o(M)$ of that scene can be rendered without resulting in underexposure. For the group of scenes measured, the maximum value of $\log R_s$ is 2.30. Hence, the exposure incident on the negative material in the camera at the point where the scene element having the luminance $B_o(M)$ is imaged is given by

$$\begin{aligned}\log E_x &= \log E_{0.3\bar{G}} + (\log R_s \text{ max} - 0.60)\\ &= \log E_{0.3\bar{G}} + 1.7.\end{aligned}$$

The perpendicular line M is drawn from this point (Fig. 17) on the $\log E_x$ scale. The horizontal line 12 represents this scene. Formula (20a) may be modified to give camera exposure when using $B_o(M)$ in this manner,

$$t/f^2 = R_s \text{ max}/4B_o(M) \cdot K \cdot S. \qquad (20d)$$

The line 13 shows how the average scene is rendered by using this criterion. The minimum value of $\log R_s$ is 0.93 and the line 14 shows how such a scene is rendered.

To determine whether or not this method will result in overexposure in some cases (the rendition of the maximum object luminance at points of insufficient gradient on the shoulder of the D_x-$\log E_x$ curve), it is necessary to examine the result when R_h has its maximum probable value. For the scenes on which measurements are available, this is found (Fig. 17) to be 1.2 log units. In this case the maximum B_o will be rendered at a point on the D_x-$\log E_x$ curve where the exposure is given by

$$\begin{aligned}\log E_x &= \log E_{0.3\bar{G}} + 1.7 + 1.2\\ &= \log E_{0.3\bar{G}} + 2.9.\end{aligned}$$

The rendition of this scene is shown by line 15. The maximum object luminance lies 1.2 log units to the right of the line M, and hence 0.5 log units to the right of the boundary of the graph sheet. In modern negative materials, the point on the shoulder of the characteristic curve having insufficient gradient to produce a high-quality negative lies at approximately $\log E_x = \log E_{0.3\bar{G}} + 2.8$. Thus, this point on the shoulder is situated approximately 0.4 log units to the right of the boundary line of Fig. 17, and hence 0.1 log unit of the tonal scale of this scene is rendered on the shoulder to the right of this point, resulting in a slight loss of highlight detail.

By the use of $B_o(M)$ in this manner, the average scene receives 4.5 times the camera exposure necessary for the production of an excellent print and the scene having minimum value of R_s 23 times that which is necessary.

Some workers may contend that in using $B_o(M)$ as a criterion of camera exposure, the line M should be located farther to the left than shown in Fig. 17, so that the minimum B_o of the average scene is rendered on the line P. An examination of the values of R_s for the entire group of 126 scenes shows that if this is done, 50 percent of the scenes will not receive sufficient exposure for the production of an excellent print.

Apart from considerations of expedience or of compromise with fundamentally sound principles because of the difficulties encountered in practical application of these principles, we are forced to conclude that, assuming proper usage, B_o min *is* the best criterion of correct camera exposure for all negative-positive processes. Its use in Eq. (20b) leads to the least camera exposure which will produce a negative from which an excellent print can be made. If any other of the criteria is applied in a manner such as to insure *sufficient* exposure in *all* cases, it leads to more than is necessary in *many* cases. For some scenes, this excess may cause sufficient overexposure to result in some loss of highlight detail. Camera exposures much in excess of those necessary give undesirably high minimum negative densities, tend to increase the graininess of the silver deposits, and may result in poor definition, because of camera or object movement or the use of diaphragm apertures larger than necessary, thus decreasing depth of focus. The use of B_o min yields negatives which are relatively uniform with respect to minimum density. This is a distinct advantage in the printing operation since it has been shown[42] that the minimum negative density is the best criterion of correct printing exposure. It is concluded that of the possible criteria which are *unique* and *measurable*, minimum object luminance is the most significant, and hence most perfectly meets the specification of a useful criterion. This being the case, an effort must be made to choose from the possible methods of evaluating the illuminating power of solar radiation that one which most precisely predicts how much light is available for the illumination of scene elements of lowest luminance, those forming the extreme shadow region of the tonal scale.

D. Determinants of Minimum Object Luminance

All three of the methods thus far considered for evaluating the light arriving at the earth's surface from the sun have certain characteristics in common. One of these is that they only give information regarding the amount of light incident upon surfaces so situated with respect to other terrestrial objects that they receive direct sunlight and by light coming directly from large expanses of the sky, varying from a maximum of 2π steradians for the horizontal plane, to a minimum of 1π steradians in the case of the perpendicular plane. Such surfaces, even if they have relatively low reflectance values, have luminances which are high compared with other surfaces so situated within the scene that they do not receive direct sunlight or light directly from very large expanses of the sky, in general, form the highlights, i.e., maximum luminances of the scene.

The surface elements of lowest luminance are seldom illuminated by direct sunlight, nor are they frequently illuminated by light coming directly from large areas (great solid angles) of the sky. In fact, they are usually illuminated either by (a) sunlight which has been reflected at least once and frequently many times from nearby objects, or (b) by light coming directly from small areas of sky, or (c) by mixtures of (a) and (b) in various proportions. It seems unreasonable to expect these methods of evaluating solar light to give a *direct* indication of the amount of light available for illuminating the surface elements of minimum luminance and it is doubtful that they are good *indirect* indices of minimum scene luminance.

Another characteristic common to these three methods is that the evaluation of the available light is in terms of a single plane in one case of fixed orientation (horizontal plane), in another (perpendicular plane) the normal to the surface remains in a horizontal position but its azimuth is variable, while in the third (normal plane) the position of the normal to the plane varies with respect to both azimuth and inclination to the plane of the horizon. To decide which one of these methods is most appropriate or whether any one of them constitutes a satisfactory criterion of how much light is available for illuminating the surface elements of lowest luminance, consideration must be given to the spatial distribution and orientation of the scene elements with respect to the *direction* (or directions) *of the incident illuminance* and the *line of sight*, which of course is the optical axis of the camera objective.

In terrestrial photography (the making of pictures of objects on the earth's surface from points on the earth's surface), the line of sight is approximately parallel to the horizontal plane. Large deviations from this condition may occur in some cases, but the most probable average direction deviates but little from the horizontal.

For the time being, we shall consider the case of a *clear atmosphere* and *front-lighted* scenes. The sun, the source of shadow-forming illumination, is approximately "behind" the camera. The azimuth of the line of sight may vary by ± 50 to 60 degrees from that of the sun without introducing appreciable cross-lighting. The solar altitude may vary from 0 to about 75 degrees. It is this variation that most profoundly affects the amount of light reaching the earth's surface.

The scene elements, terrestrial objects, are located at an infinite variety of positions in the three-dimensional space in front of the camera. To a one-eyed observer (and the camera, of course "sees" only with "monocular vision"), the scene appears to consist of a plane mosaic made up of an infinitude of two-dimensional elements varying in size and shape, the plane of the mosaic being perpendicular to the line of sight.[p] Actually, these surface elements are oriented in all possible directions. On the average, there are just as many surface elements facing in any one direction as in every other possible direction. Moreover, it is probable that the sum of the areas of all surface elements facing in one direction is the same as the sum of the areas of all of the surface elements facing in any one of every other possible direction. No *predominant* orientation of surface elements, therefore, can be assumed. For predicting the correct camera exposure in terms of the amount of light reaching the earth's surface, it seems futile to attempt to select a plane with some particular orientation as being more significant than another plane having one of many possible orientations.

However, there is a predominant *projected area* attributable to a particular scene element orientation. When an average scene (one in which the surface elements follow the probability law with respect to the distribution of areas and orientations) is viewed from a particular point in space, specifically the camera position, all surface elements are seen as projected upon a plane perpendicular to the line of sight. All surface elements not perpendicular to the line of sight are foreshortened so that their apparent areas are lessened by the amount depending upon the cosine of the angle between the normal and the line of sight. If equal total areas are assumed for all possible orientations, the maximum projected area for any single orientation will be that of the surface elements perpendicular to the line of sight. This is a perpendicular plane *facing the camera*. For front-lighted scenes, this corresponds approximately to the perpendicular plane facing the sun. This situation might be interpreted as indicating the maximum significance of the illuminance evaluated in this manner as a criterion of camera exposure. However, an inspection of curve I_{pt} (Fig. 15) shows that this value for a solar altitude of 90 degrees is 615 foot-candles. This increases rapidly, reaching a maximum of 7870 foot-candles at a solar altitude of 33 degrees, thus calling for a camera exposure at noon 13 times as great as that at about 3:45 o'clock (latitude 0 degrees in March or September). This is a violent contradiction of the known facts of practical experience. This criterion may be discarded for predicting the amount of light available for the illumination of the scene elements of minimum luminance. Incidentally, an inspection of this same curve leads to the conclusion that solar illuminance evaluated in this manner is of little significance as an indication of the maximum scene luminance.

If we were interested in choosing a criterion of maximum scene luminance, B_o max, an inspection of Fig. 15 indicates that the illuminance on the normal plane is most promising.

Attention should be called to one aspect of this situation which has not been mentioned. A surface which completely scatters or diffuses the light reflected therefrom has a constant luminance, regardless of the viewing direction. In practice, many surfaces approximate quite closely this condition. There are also many surfaces which are more or less glossy and exhibit specular (mirror-like) reflectance to a greater or lesser degree. Assuming that all surface elements composing a scene are diffuse reflectors, then it is evident from an inspection of Fig. 15 that the illuminance on the normal plane will be the best criterion of maximum scene luminance.

If it can be assumed that, on the average, a scene is composed of surface elements, the orientations and areas of which are in accordance with the probability law (and no other assumption seems tenable), it is reasonable to conclude also that those surface elements, so situated within the scene structure that they reflect light to the areas of minimum luminance, must also have the same random distribution with respect to orientation and area. This being the case, there is just as much chance that light arriving at the

[p] The human observer, even the one-eyed observer, by virtue of certain clues, such as focal accommodation, retinal image sizes, stored memories of past experiences, etc., perceives the existence of depth elements in addition to the two-dimensional surface elements. By virtue of these clues he is aware to some extent of the three-dimensional characteristics of the scene. Only a part of these clues can be reproduced photographically. The human observer when looking at a scene is aware of the directional characteristics of the light which illuminates the scene. Variations in the luminance of various surface elements are interpreted in many cases as indicative of the distance and of the orientations of these surfaces. This contributes somewhat to the perception of the three-dimensional spatial aspects of the scene. The perfect reproduction of luminance and luminance differences by the photographic process therefore is a contribution to the "quality" of the photograph probably of equal importance to the correct reproduction of the geometrical or perspective characteristics.

earth's surface from one particular direction will be reflected either singly or multiply so as to illuminate the elements of minimum luminance, as that light coming from any or every possible other direction will be so reflected. Hence, a method of evaluating the amount of light which is entirely *independent of direction of arrival* should constitute the most significant criterion of the amount of light available for the illumination of the shadow regions and therefore should be the most significant determinant of minimum luminance. The evaluation of light in terms of the *volume density of luminous energy* meets these requirements.

We may summarize the discussion presented in this section as follows:

1. The illuminance on a plane exposed to direct sunlight and to light from large expanses of the sky is not the most significant index of the amount of light available for the illumination of the surface elements of minimum luminance.

2. There is little justification for choosing a plane of any particular orientation for the evaluation of the amount of light available for photographic purposes.

3. Because of the random orientation of the surface elements which reflect light to the shadow regions of a scene, light arriving from any one direction is just as effective as that arriving from any other possible direction for the illumination of the surface elements of minimum luminance.

4. The most significant method of evaluating the useful light is one which gives equal weight to the sunlight and skylight arriving at the earth's surface from all possible directions, and hence is independent of directional aspects.

5. Such an evaluation may be obtained by using the *volume density of luminous energy* which is expressed in terms of the *total amount of light, luminous energy, Q*, in a unit volume of space at the earth's surface, regardless of the direction from which it arrives.

E. Luminous Density

The term *luminous density, V*, has been adopted in lieu of the longer phrase, volume density of luminous energy, to indicate the amount of light arriving at the earth's surface. We suggest, and will use in this communication, the symbol V to designate *volume* density of luminous energy, or, more briefly, luminous density. The defining equation is:

$$V = dQ/dv.$$

This is not a new concept, but it has not been used to any great extent (as least in this country) in illuminating engineering and its related sciences. The concept, ***luminous energy, Q***, may also be unfamiliar to some readers although it has been precisely defined. It has been proposed in the report of the Colorimetry Committee of the Optical Society of America[5] as a term worthy of definition. Such a course already seems to have been justified because the use of luminous density appears to be the most significant method of evaluating direct sunlight and skylight for the determination of correct camera exposure.

This concept of luminous density or "space density of light" has been given considerable attention abroad. Analogous quantities, such as "space illumination," "mean spherical illumination," and "mean hemispherical illumination," have been discussed at length by Gershun.[43] He attributes the introduction of the concept of space illuminations, "Raumbeleuchtung," to Weber,[44] although Moon[45] attributes its introduction to Arndt. In any case, Arndt[46] has devoted considerable attention to its development and to the application of the general concept. Gershun lays considerable stress on the advantages of using the mean spherical or hemispherical illumination (illuminance) instead of the illuminance evaluated on a plane of some particular orientation in many problems in illuminating engineering. He says, "The value of space illumination at each point in a room lighted by luminaires that throw the light in comparatively narrow beams is conveniently divided into two parts—the first produced by direct light from the luminaires, and the second produced by light reflected from surfaces in the room. The ratio of these two parts characterizes approximately the degree of direct lighting and the depth of the shadows. The value of the second part gives the average level of brightness of the enclosing surfaces and of the surfaces of objects contained within the room. . . . The actual evaluation of the adequacy and uniformity of lighting as it appears to the eye does not correspond to the evaluation of lighting based on the illumination distribution on a fictitious working surface, but is more closely characterized by the space illumination. The transition from standardization according to horizontal illumination to standardization according to space illumination is completely analogous to the change from candlepower rating to lumen rating—a change that is universally accepted in the specification of lamps."

These same arguments can be applied also to conditions outdoors where the light entering the scene space is composed of two components, direct sunlight and skylight. Thus, Gershun's statement may be paraphrased to fit the conditions existing in exterior scenes: The value of space illuminance at a point on the earth's surface may be conveniently divided into two parts, the first produced by direct light from a luminaire, the sun, and the second attributable to the light coming to that point from the entire sky hemisphere subtending a solid angle of 2π steradians, the sky being equivalent in effect to the light reflected from the walls and ceiling of a room. Luminous density as it is used in this communication is determined entirely by the sunlight and skylight *arriving* at the surface of the earth and does not include light reflected by the earth's surface or by other terrestrial objects.

The psychophysical unit of luminous energy, Q, the *lumerg*, corresponds to the physical unit of mechanical or radiometric energy, the *erg*, in the same manner that the unit of luminous flux, the *lumen*, corresponds to the mechanical unit of power or the radiometric unit of flux, the *watt*. Thus, as one watt is equal to ten million (10^7) ergs per second, so the lumerg is defined by the relation that one lumen is equal to ten million (10^7) lumergs per second.

TABLE VII. Values of illuminance (foot candles) on the normal plane and of luminous density (lumerg/cu. ft.) attributable to direct sunlight, skylight, and the total.

Solar altitude h	Air mass m	Direct sunlight Illuminance I_{nd} ft.-c	Direct sunlight Luminous density V_d ft.-lg	Skylight Illuminance I_{ns} ft.-c	Skylight Luminous density V_s ft.-lg	Total Illuminance I_{nt} ft.-c	Total Luminous density V_t ft.-lg
3	15.36	375	3.82	605	5.46	980	9.28
5	10.39	1150	11.7	767	7.96	1920	19.7
7	7.77	2070	21.1	873	10.4	2940	31.5
10	5.60	3400	34.7	987	13.7	4390	48.4
15	3.82	5080	51.8	1120	18.4	6200	70.2
20	2.90	6240	63.6	1220	21.7	7460	85.3
25	2.36	7050	71.9	1280	24.0	8330	95.9
30	2.00	7640	77.9	1320	25.5	8960	103
35	1.74	8110	82.7	1350	26.7	9460	109
40	1.55	8470	86.4	1370	27.5	9840	114
45	1.41	8720	88.9	1390	28.3	10110	117
50	1.30	8940	91.2	1400	28.8	10340	120
55	1.22	9100	92.8	1420	29.2	10520	122
60	1.15	9240	94.2	1430	29.5	10670	124
65	1.10	9350	95.4	1440	29.7	10790	125
70	1.06	9430	96.2	1450	29.9	10880	126
75	1.04	9480	96.7	1450	30.0	10930	127
80	1.02	9520	97.1	1460	30.2	10980	127
85	1.01	9550	97.4	1470	30.3	11020	128
90	1.00	9570	97.6	1480	30.4	11050	128

In this communication, luminous density is expressed as lumergs per cubic foot. The English unit of volume is used for the sake of consistency with established American usage, by which illuminance is specified in terms of lumens per square foot (*foot candles*, ft. c) and luminance by apparent lumens per square foot (*foot lamberts*, ft. L). Recognizing the convenience of a short suggestive name and abbreviation (with no regard for mathematical or dimensional logic), we suggest the term *foot lumerg* (ft. lg) to denote the English unit of luminous density, lumergs per cubic foot.

The luminous density, V, corresponding to one foot-candle of *normally* incident light can be determined from the definitions of the foot candle as one lumen per square foot. By the definition of the lumerg given in the preceding paragraph, one foot candle of *normally* incident light, which is equal to one lumen per square foot, also is equal to 10^7 lumergs per second per square foot. Since the velocity of light is $9.836 \cdot 10^8$ feet per second, one foot candle corresponds to 10^7 lumergs contained in a volume one square foot in cross section and $9.836 \cdot 10^8$ feet long. The luminous density corresponding to one foot candle (incident normally) is therefore 10^7 divided by $9.836 \cdot 10^8$, or 0.01017 lumerg per cubic foot. The values of direct sunlight illuminance, on planes normal to the rays, given in column 3 of Table VII, have been converted to lumergs per cubic foot and the results are shown in the fourth column. When light such as that from the sky is incident from many different directions, the corresponding contributions to the total luminous density may be determined by measuring or computing the illuminance incident normally on a plane perpendicular to each direction of the incident rays. Using Kimball's average data of sky luminance (Table III), the luminous density values for various solar altitudes were computed by using a technique similar to that employed for computing the illuminance on planes of specified orientation (see Section III, Eqs. 23 and 24) except that the cosine α term becomes unity, $\alpha=0$. The value of luminous density at the earth's surface due to the entire sky hemisphere on a clear day is given by the expression

$$V_s=\left(\omega_1\frac{B_{s1}}{\pi}+\omega_2\frac{B_{s2}}{\pi}+\cdots\omega_n\frac{B_{sn}}{\pi}\right)\cdot 0.01017, \quad (25)$$

where ω_1, ω_2, $\cdots\omega_n$ represent the small equal solid angular elements into which the entire hemisphere of the sky must be divided in order that the contribution of each to the total luminous density may be computed, and B_{s1}, B_{s2}, $\cdots B_{sn}$ represent the luminance (ft. L) of the corresponding portions of the sky. This method is necessary because of the non-uniformity of the luminance of the clear sky.[q]

Luminous density, as distinguished from the illuminance on the plane of some specified orientation, is independent of the direction from which the light arrives at the earth's surface. It is a measure of the total amount of luminous energy in a space of unit volume and obviously this must be independent of the direction from which the light enters this space.

In Table VII the computed values of luminous density attributable to skylight are shown in the sixth column and in the last column are shown the total sunlight plus skylight values. These data are shown graphically in Figs. 13, 14, and 15, the curves being designated as V_d, V_s, and V_t, respectively. These figures are convenient for the comparison of the various methods of evaluating the solar light arriving at the earth's surface. In Fig. 18 the luminous density attributable to direct sunlight, skylight, and the ratio of the two is shown graphically as curves V_d, V_s, and V_d/V_s, respectively. This is useful for some purposes since it shows the relative magnitudes of these two components for any solar altitude.

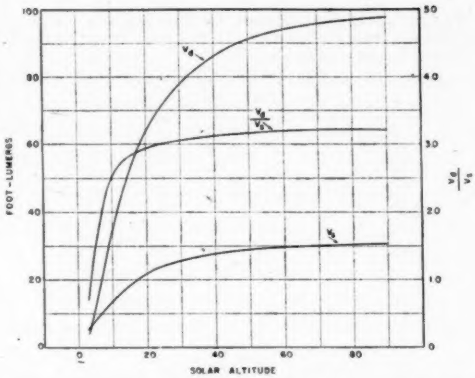


FIG. 18. Luminous density, V (foot-lumergs) for clear sky conditions as a function of solar altitude. V_d is for direct sunlight; V_s is for skylight.

Great care must be exercised in distinguishing between the nature of luminous density, which is expressed in foot lumergs, and illuminance, which is expressed in foot-candles. Values of luminous density cannot be used for computing the luminance of a surface of known reflectance because the value of V gives no information as to the direction from which the light enters the unit volume of space in terms of which it is expressed.

Objections may be raised to the use of luminous density because of the impossibility of computing or predicting the luminance of a surface therefrom. Such objections are of little significance in connection with the problem of photographic exposure because the light which ultimately is incident on the surface elements of lowest luminance may *enter* the scene space from any one or all directions. It is this very directional aspect which disqualifies all methods of evaluating the available light in terms of the illuminance on

[q] Each term in the parenthesis of Eq. (25) is the illuminance produced by only that portion of the sky represented by the solid angle ω on a surface perpendicular to the central ray of that solid angular element. For concise expression, we may call each of these quantities an *elementary normal illuminance* (see reference 46). The sum of the terms in the parenthesis of Eq. (25) is therefore the sum of all of the elementary normal illuminances and may be called the *space illuminance* (see reference 46). Gershun points out that this sum is four times the *mean spherical illuminance* and that it is the product of the velocity of radiant energy times the *luminous space density*, provided, of course, that the velocity of radiant energy is expressed in the same units of length as the volume of space. From Eq. (25) it can be concluded that the mean spherical illuminance (ft.-c) is $\frac{1}{4}\times1/0.01017=24.59$ times the luminous density (ft. lg).

It should be noted that the concept of mean spherical illuminance is applicable regardless of the shape of the source and in the present case its usefulness is not affected by the restriction of the source to the celestial hemisphere. A concept of *mean hemispherical illuminance* was also discussed by Gershun but this is not directly related to either space illuminance or luminous density and is not as useful as these or mean spherical illuminance in the present discussion.

The elaboration of the concept of volume density of luminous energy, mean spherical illuminance, and mean hemispherical illuminance and their application to photographic problems, both interior and exterior, promises to yield very useful results. However, in this discussion we cannot pursue this elaboration to its conclusion.

TABLE VIII. Specification of the latitude bands for which the variations of solar altitude with hours of the day and month are computed.

Latitude	Latitude zone
0°	5°N to 5°S
10	5 to 15°N or S
20	15 to 25°N or S
30	25 to 35°N or S
40	35 to 45°N or S
50	45 to 55°N or S
60	55 to 65°N or S
70	65 to 75°N or S
80	75 to 85°N or S
90	85 to 90°N or S

some plane having a definite orientation as a significant or meaningful criterion of minimum luminance, B_o min. The ability to compute a luminance value for a surface of known reflectance and orientation is of little or no value for the computation or prediction of required camera exposure because the surface elements of minimum luminance may have any possible orientation and no predominant average orientation can be assumed to exist. Moreover, the reflectance of these surface elements is, in general, unknown, and it does not seem justifiable to assume an average value. The diffuse reflectance of such surfaces may vary from 2 percent to 90 percent, approximately. Surfaces having minimum luminances do not necessarily have low reflectances. In many cases surfaces having relatively high reflectances are so situated within the scene structure that but little light reaches them, and hence they may become elements of minimum luminance. Incidentally, the same situation exists in the highlight region. The highest luminance, B_o max, may be associated with surface elements having relatively low reflectances.

TABLE IX. Solar altitude at various hours and months at latitude 40°N.

	Hours before or after noon, local solar time							
	0	1	2	3	4	5	6	7
June	73.4	69.1	59.8	48.8	37.4	25.9	14.8	4.2
July May	70.1	66.3	57.6	46.8	35.5	24.0	12.8	
Aug. Apr.	61.3	58.4	50.9	41.1	30.1	18.7	7.2	
Sept. Mar.	50.0	47.7	41.6	32.8	22.5	11.4		
Oct. Feb.	38.7	36.8	31.6	23.9	14.5	3.9		
Nov. Jan.	29.9	28.3	23.7	16.7	8.0			
Dec.	26.6	25.1	20.7	14.0	5.5			

It has been suggested that minimum scene luminance may be satisfactorily determined by using as an index the illuminance as evaluated on the normal plane and by assuming that on the average a constant attenuation of light by successive reflectance (and possible transmittance) occurs before this light finally illuminates the scene elements of minimum luminance. Examination of the experimental data indicates that the attenuation varies over a great range and it is doubtful whether the statistical average of this attenuation factor is sufficiently significant to be useful. In groups of scenes which are relatively homogeneous in spatial structure, this attenuation tends to vary over a more limited range. By the collection of voluminous statistical data, attenuation factors for different scene types could perhaps be established of sufficient uniqueness to be useful. This procedure does not appear to have much to recommend it in preference to the use of luminous density. Moreover, the use of the normal plane illuminance values tends to imply that the highlight, B_o max, in a scene is the determinant of correct camera exposure. This is not the case and it seems preferable to use luminous density which carries no such implication. The very fact that luminous density is expressed in foot lumergs, i.e., lumergs per cubic foot, which carries no directional connotation, removes any possible temptation, to which the unwary might be subjected, to attempt the computation of luminous values of a scene element of known reflectance.

V. LIGHT INDEX AS DETERMINED BY SOLAR ALTITUDE

When the sun is unobscured, the solar altitude can be measured with simple and inexpensive devices with precision sufficient for photographic exposure purposes. This cannot be done when the sun is obscured by clouds so that there are no cast shadows. It is more convenient for practical purposes to use the time of day as an index of solar altitude. At any point on the earth's surface the solar altitude is the same at *a* hours before noon as at a hours after noon, provided noon is defined as the time when the sun is on the local meridian, that is, in terms of *local solar* time. The use of local solar time therefore simplifies the task of constructing tables for finding solar altitude. The altitude of the sun at any hour of the day is dependent upon the position on the earth at which it is observed, upon the time of day, of the year, and upon the declination of the sun. In order that the luminous density data shall be applicable to all parts of the world, the solar altitude must be computed for various hours of the day and month for several different latitudes. The latitudes for which the calculations have been made and the latitude band included are shown in Table VIII. Solar altitude at any place and time may be determined by the formula:

$$\sin \alpha = \sin \beta \sin \gamma + \cos \beta \cos \gamma \cos \text{hour angle}, \quad (26)$$

where α is solar altitude, β is the latitude of the place, and γ is the declination of the sun. The hour angle is the number of hours before or after local solar noon.

The solar altitude varies continuously from day to day but the total variation during any one-month period is relatively small. It seems sufficient, therefore, to compute values of solar altitude for the 21st of each month. Moreover, the solar altitude on the 21st of July is the same as on the 21st of May, and similarly for other pairs of months. By taking advantage of these circumstances, a simple table may be constructed showing solar altitude for the hours of the day

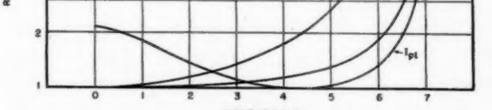


FIG. 19. Curves showing the dependence of required camera exposure (relative) upon hour angle as determined by different methods of evaluating the total available light.

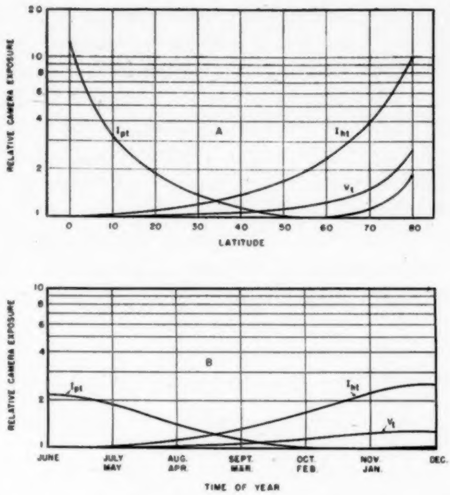


FIG. 20. A: Curves showing the dependence of required camera exposure (relative) upon latitude for various methods of evaluating the total available light. B: Curves showing the dependence of required camera exposure (relative) upon month for various methods of evaluating the total available light.

and the months of the year at any specified latitude. Table IX shows these values for latitude 40°N. The same table is valid for points 40° south of the equator, provided the month column at the left is inverted. With this table and similar ones for other selected latitudes it is possible to read from the curves in Fig. 15 values of I_{ht}, I_{pt}, I_{nt}, and V_t, for any hour, month, and latitude.

In the previous section we have discussed various methods of evaluating the direct sunlight and skylight arriving at the earth's surface and have presented arguments, which in our opinion are convincing, supporting the use of luminous density as the most rational criterion of minimum scene luminance, B_o min. Before proceeding further it seems desirable to present some quantitative data in support of these conclusions. To test the comparative merits of evaluating the light arriving at the earth's surface, it is only necessary to consider relative values of camera exposure which each method of evaluating the available light will demand. In Fig. 19 are plotted the relative camera exposures derived from each of the four methods of evaluating solar light. In each case, the maximum light is considered to require a camera exposure of unity (1.0). The curves are for front-lighted scenes at 40°N latitude on June 21. If a camera exposure of unity, (1.0), is required at noon, the use of I_{ht} requires a camera exposure of 2.6 at hour angle 5 (5 hours before or after mean solar noon), of 5.5 at hour angle 6, and of 29 at hour angle 7. These increases are much greater than years of experience on the part of many workers have shown to be necessary. The use of V_t requires approximately a twofold increase of the noon exposure at hour angle 6 and eightfold at hour angle 7. I_{nt} gives increases very nearly the same as V_t except at very low solar altitudes and even there the difference is so small as to be of little moment.

TABLE X. Luminous densities at latitude 40°N.

	Hours before or after noon, local solar time							
	0	1	2	3	4	5	6	7
LUMINOUS DENSITY, FOOT-LUMERGS								
June	127	126	124	119	112	98	70	16
July May	126	125	123	118	110	94	62	
Aug. Apr.	124	123	120	115	104	82	33	
Sept. March	120	119	115	107	91	55		
Oct. Feb.	113	111	106	94	69	14		
Nov. Jan.	103	101	94	77	38			
Dec.	99	96	87	67	23			
LUMINOUS DENSITY RATIOS, R_o								
June	1.01	1.02	1.03	1.08	1.14	1.31	1.83	8.0
July May	1.02	1.02	1.04	1.09	1.16	1.36	2.06	
Aug. Apr.	1.03	1.04	1.07	1.11	1.23	1.56	3.88	
Sept. March	1.07	1.08	1.11	1.20	1.41	2.33		
Oct. Feb.	1.13	1.15	1.21	1.36	1.86	9.15		
Nov. Jan.	1.24	1.27	1.36	1.68	3.27			
Dec.	1.29	1.33	1.47	1.91	5.57			
LIGHT INDICES, L_e								
June	0	0	0	0	1	1	3	9
July May	0	0	0	0	1	1	3	
Aug. Apr.	0	0	0	0	1	2	6	
Sept. March	0	0	0	1	1	4		
Oct. Feb.	1	1	1	1	3	10		
Nov. Jan.	1	1	1	2	5			
Dec.	1	1	2	3	7			

TABLE XI. Consecutive whole numbers, column 3, which represent all values of light index, L_e, between the limits as shown in the first column and the corresponding values of luminous density, Column 4.

Luminous density ratio, R_o Limits	Midpoint	Light index L_e	Luminous density Ft.-lg
1.00– 1.12	1.00	0	128
1.13– 1.41	1.26	1	102
1.42– 1.78	1.59	2	80.5
1.79– 2.24	2.00	3	64.0
2.25– 2.83	2.52	4	50.8
2.84– 3.56	3.18	5	40.3
3.57– 4.49	4.00	6	32.0
4.50– 5.66	5.04	7	25.4
5.67– 7.13	6.35	8	20.2
7.14– 8.98	8.00	9	16.0
8.99–11.3	10.0	10	12.8

The curve V_t also represents the I_{nt} values. I_{pt} indicates that the minimum camera exposure is required at hour angle 4.3 with twice as much needed at noon and at hour angle 6.5. This very definitely disagrees with the known facts.

Figure 20A shows the variation of camera exposure with latitude at noon on March 21 (or September 21). Here the failure of I_{pt} to give reasonable results is strikingly illustrated. This criterion indicates that a minimum exposure is required at latitude 57°, increasing by a factor of 13 at latitude 0, and by a factor of 1.8 at latitude 80°. The use of I_{ht} demands twice as much camera exposure at latitude 56° as at latitudes less than 10°, and a fourfold and a tenfold increase at latitudes of 70° and 80°, respectively. Camera exposure called for by V_t increases very slowly from 0 to 40°, but rises to 1.5 at 70° and to approximately 3 at 81°. Experience indicates that, in traveling from latitudes of 50 or 60 degrees to the equator, no decrease of camera exposure at midday is permissible. In fact, some observers recommend an increase in the camera exposure as the equator is approached. This increase is not required because of a decrease in the amount of solar light arriving at the earth's surface, but rather because of directional aspects which result in "top lighting" (vertical cross-lighting) effects giving rise to depressed values of B_o min without the usual variation in the spatial aspect of scene structure.

In Fig. 20B is shown the variation in the required camera exposure throughout the year at latitude 40° at noon. At this latitude the solar altitude (at noon) decreases from 73.4° (June 21) to 26.6° (December 21). This is approximately the same decrease in solar altitude that occurs from noon, June 21, to 5 p.m. (or 7 a.m.) on the same day. Experienced photographers of outdoor subjects agree that the camera exposure required at 5 p.m. is not appreciably greater than at noon during midsummer. The solar distance is approximately 7 percent less on December 21 than on June 21. Hence, the total amount of light coming to the earth should be about 7 percent greater in December for the same solar altitude and identical atmospheric conditions. This condition tends to give more, rather than less, light with the sun at identical altitudes. Hence, there should be less change in the required camera exposure from June to December than from noon to 5 p.m. on June 21. Again, experience agrees with this conclusion. The most significant method

TABLE XII. Light indices for various latitudes, local solar time.

Latitude 0°

North	Noon	11 1	10 2	9 3	8 4	7 5	6 : 30 A.M. 5 : 30 P.M.	South
June	0	0	0	0	1	3	6	Dec.
July May	0	0	0	0	1	3	6	Jan. Nov.
Aug. Apr.	0	0	0	0	1	3	6	Feb. Oct.
Sept. Mar.	0	0	0	0	1	3	6	Mar. Sept.
Oct. Feb.	0	0	0	0	1	3	6	Apr. Aug.
Nov. Jan.	0	0	0	0	1	3	6	May July
Dec.	0	0	0	0	1	3	6	June

Latitude 10°

North	Noon	11 1	10 2	9 3	8 4	7 5	6 A.M. 6 P.M.	South
June	0	0	0	0	1	2	9	Dec.
July May	0	0	0	0	1	2	10	Jan. Nov.
Aug. Apr.	0	0	0	0	1	2		Feb Oct.
Sept. Mar.	0	0	0	0	1	3		Mar. Sept.
Oct. Feb.	0	0	0	0	1	3		Apr. Aug.
Nov. Jan.	0	0	0	1	1	4		May July
Dec.	0	0	0	1	1	4		June

Latitude 20°

North	Noon	11 1	10 2	9 3	8 4	7 5	6 A.M. 6 P.M.	South
June	0	0	0	0	1	2	5	Dec.
July May	0	0	0	0	1	2	6	Jan. Nov.
Aug. Apr.	0	0	0	0	1	2	10	Feb. Oct.
Sept. Mar.	0	0	0	0	1	3		Mar. Sept.
Oct. Feb.	0	0	0	1	1	4		Apr. Aug.
Nov. Jan.	0	0	0	1	2	7		May July
Dec.	0	0	1	1	2	8		June

TABLE XII.—*Continued.*

Latitude 30°

North	Noon	11 1	10 2	9 3	8 4	7 5	6 A.M. 6 P.M.	South
June	0	0	0	0	1	1	4	Dec.
July May	0	0	0	0	1	1	4	Jan. Nov.
Aug. Apr.	0	0	0	0	1	2	7	Feb. Oct.
Sept. Mar.	0	0	0	1	1	3		Mar. Sept.
Oct. Feb.	0	0	0	1	2	6		Apr. Aug.
Nov. Jan.	0	1	1	1	3			May July
Dec.	1	1	1	2	4			June

Latitude 40°

North	Noon	11 1	10 2	9 3	8 4	7 5	6 6	5 A.M. 7 P.M.	South
June	0	0	0	0	1	1	3	9	Dec.
July May	0	0	0	0	1	1	3		Jan. Nov.
Aug. Apr.	0	0	0	0	1	2	6		Feb. Oct.
Sept. Mar.	0	0	0	1	1	4			Mar. Sept.
Oct. Feb.	1	1	1	1	3	10			Apr. Aug.
Nov. Jan.	1	1	1	2	5				May July
Dec.	1	1	2	3	7				June

Latitude 50°

North	Noon	11 1	10 2	9 3	8 4	7 5	6 6	5 A.M. 7 P.M.	South
June	0	0	0	0	1	1	2	5	Dec.
July May	0	0	0	0	1	1	2	7	Jan. Nov.
Aug. Apr.	0	0	0	1	1	2	5		Feb. Oct.
Sept. Mar.	0	1	1	1	2	4			Mar. Sept.
Oct. Feb.	1	1	1	2	4				Apr. Aug.
Nov. Jan.	2	2	3	4					May July
Dec.	2	2	4	6					June

TABLE XII.—*Continued.*

Latitude 60°

North	Noon	11 1	10 2	9 3	8 4	7 5	6 6	5 7	4 A.M. 8 P.M.	South
June	0	0	0	0	1	1	2	3	6	Dec.
July May	0	0	0	1	1	1	2	4	10	Jan. Nov.
Aug. Apr.	0	0	1	1	1	2	4			Feb. Oct.
Sept. Mar.	1	1	1	2	3	6				Mar. Sept.
Oct. Feb.	2	2	3	4	9					Apr. Aug.
Nov. Jan.	4	5	7							May July
Dec.	6	7								June

Latitude 70°

North	Noon	11 1	10 2	9 3	8 4	7 5	6 6	5 7	4 8	3 9	2 10	1 A.M. 11 P.M.	Mid- night	South
June	0	0	0	1	1	1	2	2	3	5	7	9	10	Dec.
July May	0	1	1	1	1	1	2	3	5	7				Jan. Nov.
Aug. Apr.	1	1	1	1	2	2	4	7						Feb. Oct.
Sept. Mar.	2	2	2	3	4	8								Mar. Sept.
Oct. Feb.	5	5	7											Apr. Aug.

Latitude 80°

North	Noon	11 1	10 2	9 3	8 4	7 5	6 6	5 7	4 8	3 9	2 10	1 A.M. 11 P.M.	Mid- night	South
June	1	1	1	1	1	1	1	2	2	2	3	3	3	Dec.
July May	1	1	1	1	1	1	2	2	3	3	4	4	4	Jan. Nov.
Aug. Apr.	2	2	2	2	2	3	4	5	7	9				Feb. Oct.
Sept. Mar.	4	4	5	6	8									Mar. Sept.

Latitude 90°

North	All hours	South
June	1	Dec.
July May	2	Jan. Nov.
Aug. Apr.	4	Feb. Oct.

of evaluating the available light should therefore call for little, if any, increase in the required camera exposure at midday throughout the year at latitude 40°, assuming of course the same scene structure and direction of viewing. The relative camera exposure called for by V_t agrees quite well with this condition, that demanded in December being only 1.3 times as great as that needed in June. I_{ht} demands 2.50 times as much in December as in June. I_{pt} calls for a minimum camera exposure on about February 1 and a maximum, twice as much, on June 21. This does not agree with known facts.

From these comparisons, it is evident that the use of V_t gives relative values of correct camera exposure more consistent with experience than either I_{ht} or I_{pt}. Both from practical experience and from theoretical considerations, we conclude, therefore, that luminous density (V) is the most suitable method of evaluating the solar light arriving at the earth's surface for use in establishing the relationship between correct camera exposure and solar altitude. We have already set forth the reasons for preferring V_t to I_{nt}, even though they give almost identical results except at very low solar altitudes. There is no serious objection to the use of V_t from the standpoint of simplicity of determination. Either of these quantities can be measured directly by suitably designed instruments. If they are to be computed from the available data relating to sky luminance and to illuminance due to direct sunlight on the horizontal plane, the process of obtaining values of V_t is similar in method to that involved in the case of I_{nt}.

In view of the evidence presented in the previous pages, we have decided to use luminous density as the most significant criterion of the correct camera exposure. The values of luminous density (V_t) for various months and hour angles at latitude 40°N are shown in the first section of Table X. While luminous density is the most significant index of the amount of light available for illuminating the scene element of minimum luminance, it cannot be used, as has been explained previously, for the computation of B_o min. Its merit lies in the reliability with which it indicates ***relative magnitudes*** of B_o min for various values of solar altitude. It can be most conveniently used in the form of a ratio obtained by dividing the luminous density, 128 foot lumergs for solar altitude of 90 degrees (this is the maximum luminous density) by the specific values for other solar altitudes. These ratios are shown for latitude 40° in the second section of Table X under the heading "luminous density ratios, R_v," defined by the equation

$$R_v = V_{max}/V_a. \qquad (27)$$

To reduce the computation of camera exposure to the simplest possible terms, these relative luminous density values can be expressed in terms of light index, L_v, as shown in the third section of Table X. *Light index* is defined by the equation

$$L_v = 10 \cdot \log R_v. \quad (28)$$

For the average person, the process of addition is much simpler than that of multiplication. This is especially true if the numbers to be added can be reduced to whole numbers of not more than two digits. The addition of logarithms is equivalent to the multiplication of the numbers corresponding thereto. Hence, by taking the logarithms of the numbers in the second section of Table X, multiplying them by 10, and rounding off to the nearest whole number, the numbers desired can be obtained. The error introduced by this rounding-off process is inappreciable, being equivalent to $\pm 1/6$ of the image illuminance change produced by changing the diaphragm setting one stop, such as from $f/8$ to $f/11$. This is illustrated in Table XI, which shows the limits of the luminous density ratios which are represented by the consecutive whole numbers in the light index column.

By the same method in which the light index values were determined for latitude 40° (shown in the third section of Table X) we have computed the light index data which apply to any place on the earth's surface. The latitudes for which the calculations were made and the latitude band included are shown in Table VIII. The light index values corresponding to the ten latitudes selected are given in Table XII.

These, now, represent the fundamental data for the *clear* atmospheric condition. The way in which the amount of light reaching the earth's surface is modified by atmospheres which are not clear is discussed in the next section.

VI. ATMOSPHERIC INDEX

It should be emphasized again that, thus far, the evaluation of the amount of sunlight and skylight available at the earth's surface (see Tables IV and VII) has been based on the assumption of an atmospheric condition which, as appraised by visual inspection, is designated as *clear*. Such an atmosphere contains a small amount of condensed water vapor in the cloud zone and some dust suspended in the air relatively close to the earth's surface. With the sun at the zenith the *clear* atmosphere produces a decrease in direct sunlight illuminance on the normal plane which is only 10 percent greater than the decrease in illuminance attributed to an atmosphere entirely free of condensed water vapor and dust.

We shall now consider how the amount of light available at the earth's surface is modified by atmospheres which contain larger amounts of condensed water vapor and which, as judged by visual appearance, are not clear, that is when haze or cloud is definitely visible. The chief cause of such atmospheric non-clarity is the presence of condensed water vapor, clouds, fog, haze, ice crystals, etc. This diminishes the amount of solar light which reaches the earth and profoundly alters its directional characteristics.

By using well-known and simple instruments (photometers) the amount of available light with any particular atmospheric condition can be measured either in absolute terms or relative to a clear atmosphere. The primary purpose of constructing exposure tables is to obviate the necessity of using instruments for the determination of correct photographic exposure, either for the sake of greater convenience in field work or because the instruments are unavailable. We are faced, therefore, with the task of composing verbal descriptions or specifications of certain atmospheric conditions for which the amount of available light bears a known relationship to that of a clear atmosphere, the reference condition. The

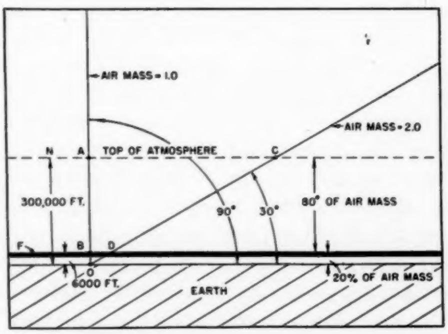


FIG. 21. Diagram illustrating the relation between air mass and solar altitude.

validity of such relationships can rest only on photometric measurements or upon extensive photographic experience. Because the atmospheric conditions encountered are exceedingly diverse and complex, and because verbal language (as contrasted with numerical specifications) is deplorably inadequate for the conveyance of *precise* ideas, the task is by no means easy. An elementary discussion of what happens to the light coming to the earth from the sun when the atmosphere contains condensed water vapor may assist therefore in the establishment of more definite concepts.

First of all, fogs and clouds must be clearly distinguished because they produce entirely different effects upon photographic exposure and so must be treated separately. According to Humphreys[47] the condensation of water vapor in the atmosphere "is divided primarily into fog and cloud, but a sharp distinction between them, that would enable one always to say which is which, is not possible. In general, however, a fog differs from a cloud only in its location. Both are owing, as explained, to the cooling of the atmosphere to a temperature below its dew point, but in the case of the cloud this cooling usually results from vertical convection, and, hence, the cloud is nearly always separated from the earth, except on mountain tops. Fog, on the other hand, is induced by relatively low temperatures at, or near, the surface, and, commonly, itself extends quite to the surface, at least during the stage of its development. In short, fog consists of water droplets or ice spicules, condensed from, and floating in the air near the surface; cloud, of water droplets, or ice spicules, condensed from, and floating in, the air well above the surface. Fog is a cloud on the earth; cloud a fog in the sky." Thus, it is apparent that clouds determine how much solar light (direct sunlight plus skylight) reaches the earth's surface. Since fogs occupy the space between the camera and the scene elements, they control not only the illumination of the scene but also determine how much of the light reflected from the scene reaches the camera. Moreover, light scattered by fog particles in the space between camera and scene elements modifies not only the object luminance, B_o, but also the luminance differences, ΔB_o, as seen from the camera position.

A. Clouds

Clouds are formed at elevations above the earth varying from a few hundred to many thousand feet. Compared with the vertical extension of the atmosphere, they generally lie relatively close to the earth. Insofar as the luminous density at the earth's surface is concerned, we may assume that clouds occupy a rather thin layer or zone extending from an elevation of approximately 3000 feet to 6000 feet. This layer we shall refer to as the *cloud zone*, which is represented in Fig. 21 by the heavy black line, *F*.

In order to simplify certain computations which we shall make concerning the amount of light reaching the earth's surface for various atmospheric conditions, we shall assume that, for any cloud condition, all of the *condensed* water vapor in the atmosphere lies within the cloud zone. This assumption implies that the atmosphere between the earth's surface and the cloud zone contains no *condensed* water vapor and also that the atmosphere above the cloud zone contains no *condensed* water vapor. Thus, all of the scattering and absorption of light which can be ascribed to *condensed water vapor*, that is, cloud or haze, takes place within the cloud zone.

Since about 80 percent of the total air mass lies above the cloud zone, the direct solar illuminance incident on the upper surface of the cloud zone is 20 percent greater than that incident on the earth's surface when the atmosphere contains no condensed water vapor and no dust. According to Moon's computations, the illuminance on the normal plane just outside the earth's atmosphere is 12,000 foot-candles, which is equivalent to a luminous density of 122 foot-lumergs.[r] The visual transmittance of one air mass of perfectly clear atmosphere[s] is 90 percent. This value is derived from spectroradiometric measurements made at Mount Whitney[16] and corrected for sea level. The luminous density at the earth's surface due to

[r] The total luminous density at the earth's surface with the sun at the zenith has previously been given as 128 foot-lumergs (as shown in Table VII) which is somewhat higher than the 122 foot-lumergs at the upper boundary of the atmosphere. No paradox is involved, however, in the fact that the luminous density at the earth's surface exceeds the amount outside the atmosphere as there is no principle of conservation of *energy density*.

[s] The term *perfectly clear* is used to designate an atmosphere which contains no *condensed* water vapor and no dust.

direct sunlight is therefore 110 foot-lumergs, the loss due almost entirely to molecular scattering being 12 foot-lumergs. Eighty percent of this is loss by molecular scattering along path AB (Fig. 21), and hence at B the value of luminous density due to direct sunlight is 112 foot-lumergs. More important than the absolute value of luminous density at B is the fact that this varies with solar altitude just as it does at the earth's surface. This is illustrated in Fig. 21, in which the line OA represents the length of path in the atmosphere traversed by direct sunlight for a solar altitude of 90 degrees and an air mass of 1.0. The line AB is the atmospheric path length to the upper surface of the cloud zone, the air mass traversed being 0.80. The line CO is the path length when the solar altitude is 30 degrees, this being equivalent to an air mass of 2.0, while the line CD is the corresponding path length to the cloud zone which, expressed in terms of air mass, is 1.60. Since loss by molecular scattering is directly proportional to air mass traversed, it is obvious from Fig. 21 (keeping in mind the assumptions we have made concerning the concentration of the condensed water vapor in the cloud zone) that the relation between solar altitude and the luminous density due to direct sunlight is the same at the upper surface of the cloud zone as it is at the earth's surface.

Is the same relationship valid for luminous density attributable to skylight? Since only 80 percent of the air mass lies above the cloud zone, the loss of light due to molecular scattering by the atmosphere between A and B will therefore be less than along the path AO. Hence, the luminous density due to skylight at the upper surface of the cloud zone will be only 80 percent of that due to the same source at the earth's surface. From Table VII it will be seen that for a solar altitude of 90 degrees the luminous density due to skylight, B_s, at the earth's surface is 30 foot-lumergs. This value, however, is derived from an average of measurements made when some condensed water vapor and dust were present corresponding approximately to the average condition on "clear" days at Washington, D. C. For direct sunlight, the visual transmittance of one air mass under these conditions is 80 percent rather than 90 percent for an atmosphere containing no condensed water vapor and no dust. The value of 30 lumergs attributable to skylight can therefore be assumed to be somewhat higher than, perhaps double, that which should be properly assigned to an atmosphere containing no condensed water vapor or dust. Using this ratio as an approximate correction factor, 15 lumergs is attributable to skylight. Eighty percent of this value is 12 foot-lumergs, this being the value of luminous density due to sunlight at the upper boundary of the cloud zone. Moreover, and more important than the absolute values, the relation between solar altitude and luminous density due to skylight at the upper surface of the cloud zone is the same as that between solar altitude and luminous density at the earth's surface.

Since the relation between solar altitude and luminous density at B attributable to both sunlight and skylight is the same (at least approximately) as at O, the values of R_v and L_v in Table X are valid indices of the relative values of *total* luminous density at the upper surface of the cloud zone for various solar altitudes.

The problem now confronting us is to find, for a constant solar altitude, the relative values of V (at O) for a clear sky, and V_t' (also at O) for various amounts of condensed water vapor in the cloud zone. Let the ratio V_t/V_t' be designated as R_a. Then the atmospheric index A is given by

$$A = 10 \cdot \log R_a. \qquad (29)$$

Meteorologists classify clouds according to their elevation, form, and structure. This classification is of little use for our purpose since the modification of the solar light is more dependent upon the position of the cloud masses in the sky relative to that of the sun and upon the extent and density of the formation than on the type of formation.

Rather extensive studies of cloudy sky luminance and of the illuminance at the earth's surface due to cloudy or hazy skies have been made, especially by the meteorologists. One of the most extensive of such investigations is that carried out by Kimball[40] and his co-workers at Washington, D. C., and Chicago, Illinois. Some 85,000 readings of sky brightness were made in the course of their work. From these data the corresponding luminous densities may be computed. We have examined this material very carefully and our conclusions as to the relative luminous

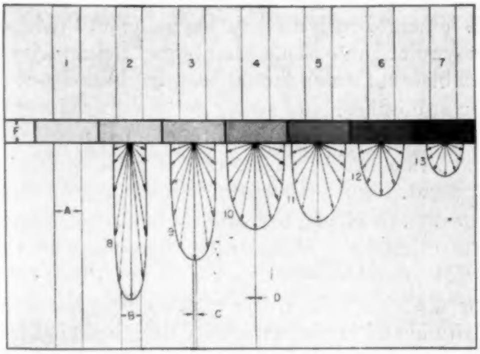


Fig. 22. Qualitative representative of the intensity and distribution of sunlight after passing through the cloud zone containing various amounts of condensed water vapor.

densities under various cloudy and hazy sky conditions have been reached by giving considerable weight to these data.

During the last three or four years, an automatic recording instrument calibrated to read directly in luminous density (foot-lumergs) has been in operation in these laboratories. Some seven or eight hundred complete daily records showing the variation in luminous density from dawn to dusk have been accumulated. At the same time, photographs in Kodachrome of the whole sky hemisphere have been made on many of the same days during which the luminous density measurements were made. These photographs were taken at one minute intervals and an inspection of these pictures has enabled us to obtain a rather definite correlation between sky appearance and the corresponding luminous density. This program has provided a mass of data concerning the relation between atmospheric conditions varying from the clear condition through the whole gamut of change (light, medium, and heavy haze; light, medium, and dense clouds; etc.) and the luminous density arriving at the earth's surface.[4]

The relative values of luminous density assigned to the atmospheric conditions described in the following pages are based upon a careful analysis and weighting of all of the various data, including our own, those obtained by Kimball (loc. cit.), and some from other sources.

Figure 22 illustrates qualitatively the intensity and distribution of the solar light after passing through the cloud zone containing various amounts of condensed water vapor. The horizontal strip F represents the cloud zone. At 1 is shown what happens when a beam of sunlight passes through the cloud zone when no condensed water vapor or dust is present, a condition that is denoted by the term *perfectly clear*. It proceeds directly to the earth's surface with no diminution in intensity except that caused by the molecular scattering which occurs between the cloud zone and the earth. This scattered light added to that which was scattered in the atmosphere before reaching the cloud zone (A to B in Fig. 21) constitutes the skylight component of total solar luminous density.

The difference between the atmospheric condition which, as appraised by visual inspection, is designated as "clear," and that of the *perfectly clear* atmosphere is so small that it cannot be conveniently illustrated graphically in Fig. 22. As will be seen by referring to Table XIII, the loss in the amount of direct sunlight, V_d, reaching the earth's surface, due to scattering by the condensed water vapor and dust associated with the "clear" condition, is only 10 percent. It may be considered, therefore, that in Fig. 22 the diagram designated as 1 represents approximately the "clear" as well as the *perfectly clear* condition. It is interesting to note also (see Table XIII) that the luminous density attributable to skylight, V_s, is about twice as great for the "clear" as for the *perfectly clear* condition. As a result, the total luminous density, V_t, is slightly greater for the so-called clear atmosphere than for one containing no condensed water vapor or dust.

Approximately Uniform Haze and Cloud Conditions

In addition to the "clear" atmospheric condition, there are five or six different haze and cloud conditions that can be distinguished and described so that they can be recognized fairly readily by visual observation.

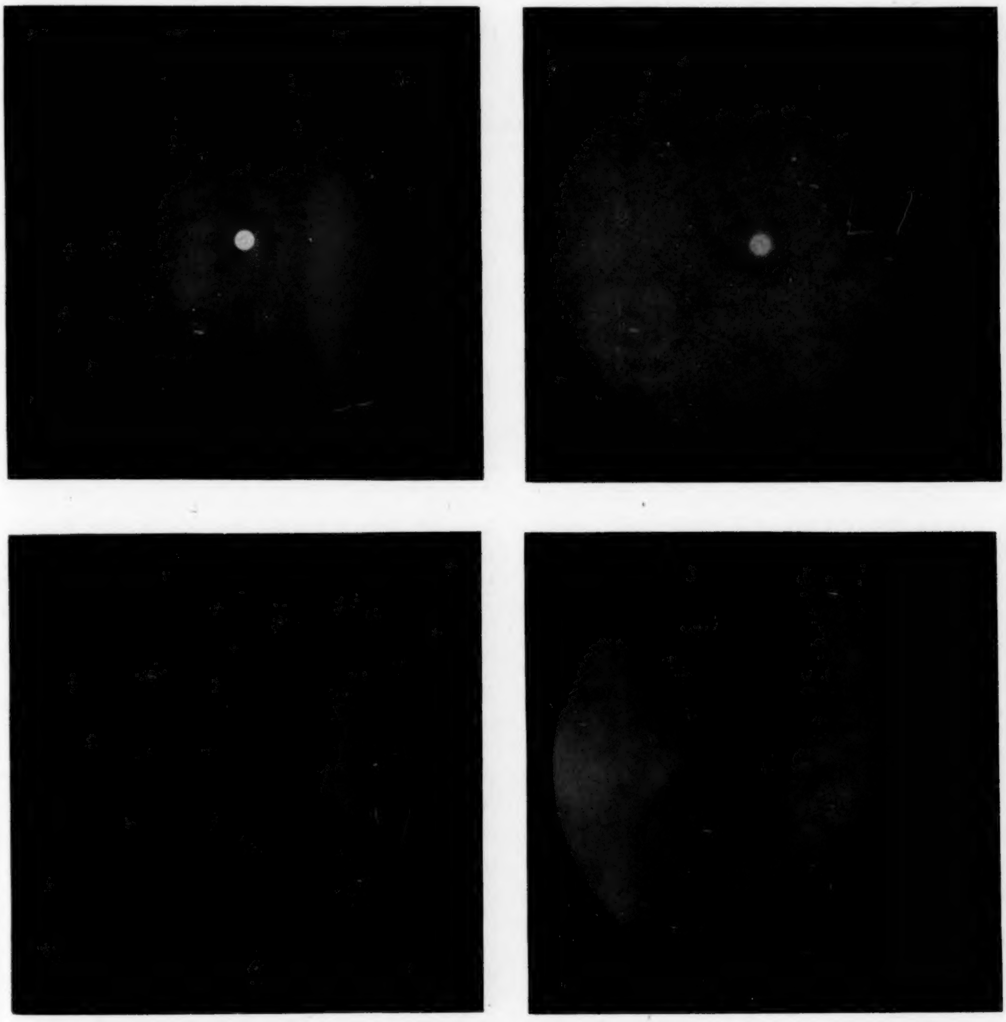
1. Light haze.—We shall consider first what happens when the condensation takes place uniformly throughout an area so great that the

[4] A more complete account of this work, together with the detailed information derived therefrom, will be published in the near future.

resultant cloud or haze appears to fill the entire sky, stretching to the horizon in every direction. The first visual indication of this condensation process is an increase in the luminance of the sky accompanied by a loss in saturation of the blue sky color. As condensation progresses, a point is reached where the blue of the sky has completely disappeared and is replaced by white which is much brighter than the clear blue sky. This is especially true in the vicinity of the sun where the sky luminance is so high as to be almost glaring or dazzling. These uniformly distributed thin cloud layers are generally called *haze*, the one just referred to being termed *light haze*.

When the sun is at the zenith ($m = 1.0$), the luminance of the sun's disk with a *perfectly clear*

A B



D C

FIG. 23. Pictorial representation of four atmospheric conditions:

A. Clear.
B. Medium Haze.
C. Medium Clouds.
D. Heavy Clouds.

TABLE XIII. Numerical values which characterize the atmospheric conditions indicated in Column 1.

Atmospheric conditions	Sun B	Sky illuminance foot-lamberts Max.	Avg.	Min.	Luminance density foot-lumergs V_d	V_s	V_t	Rel. camera exposure R_e	Atmospheric index A
Perfectly clear	**500×10^6**	(3) 1,400	740	(2) 400	**110**	15	125	—	—
Clear	**450×10^6**	(1) **6,000**	**1500**	(4) **650**	**97.6**	**30.4**	**128**	1.0	0
Light haze	14×10^6	(1) 100,000	4300	(4) 1800	3.1	87.9	91	1.4	1.5
Medium haze	450×10^3	(1) 25,000	3150	1500	0.097	63.9	64	2.0	3
Heavy haze	15×10^3	(1) 8,000	2150	1200	0.003	44	44 }	3.2	5
Light cloud	0	(1) 4,000	1750	1000	0	36	36 }		
Medium cloud	0	(2) 2,500	1300	(3) 650	0	26	26	5	7
Heavy cloud	0	(2) 1,200	640	(3) 400	0	13	13	10	10
Dense cloud	0	(2) 400	200	(3) 150	0	4	4	32	15

(1) near sun
(2) near zenith
(3) near horizon
(4) approximately 90° from sun, measured through zenith.

atmosphere is approximately 500,000,000 foot lamberts. This decreases slowly with increasing air mass, approximately 10 percent for each additional unit of air mass. For $m=5$ (solar altitude, 11 degrees), the luminance of the sun is still 330,000,000 foot lamberts. The solar altitude at noon, on June 26, for latitude 60 degrees north is 53.4 degrees ($m=1.24$), hence the luminance of the sun's disk is only about 2.7 percent less than the zenith value, approximately 487×10^6 foot-lamberts.

The luminance of the sun decreases very rapidly as condensation progresses. For the average atmosphere (at Washington, D. C., for an entire year) which is appraised by visual inspection to be "clear," the luminance of the solar disk (for solar altitudes greater than 50 degrees) is approximately 450×10^6 foot lamberts. Thus, there is a 10 percent loss in direct solar light when the amount of condensed water vapor is, on the average, visually imperceptible. This condition we propose to take as the *practical* maximum in terms of which to express the amount of direct solar light available for atmospheric conditions resulting from larger quantities of condensed water vapor. This seems reasonable since the *perfectly clear* condition is rare in many parts of the world and, even when it does occur, the increase in available luminous energy, V_d, due to direct sunlight, is only 10 percent, while that attributable to skylight, V_s, must be somewhat less. So far as we have been able to determine, no measurements of sky luminance have been made for the *perfectly clear* condition. In any case, the total luminous density, V_t, available at the earth's surface, as derived from Moon's proposed standard curve and the Weather Bureau's Washington measurements, approximates closely enough to the maximum obtainable to be satisfactory for all practical photographic purposes.

When the *light haze* condition is reached, the luminance of the sun has dropped to about 14×10^6 foot-lamberts. Thus, the light haze layer has a specular density of approximately 1.5. This decrease in the luminance of the sun is not visually apparent because the sensitivity of the human eye to luminance changes at this extremely high level is very low, and even with light haze the sun's disk is so bright that it cannot be viewed directly without severe physical discomfort and possible damage to the retina. The cast shadows are still very distinct, although not as "black" in appearance as those observed under the clear atmospheric condition.

The light haze condition is illustrated at 2 in Fig. 22, where a beam of sunlight is incident on the cloud zone containing some condensed water vapor. After traversing this zone, the light has been separated into two components, one, the direct unscattered component which is somewhat less in intensity than the incident beam, as indicated by the narrower width at B. The other, the diffuse component, is indicated by the semi-elliptical figure, 8. The lengths of the vectors drawn from a point on the lower boundary of the cloud zone indicate qualitatively the intensity of the scattered light in each of the vectorial directions. This indicates that the sky in the immediate vicinity of the sun is very bright and that this luminance falls off rapidly as the angular distance from the sun increases.

The major axis of the semiellipse 8 and also of the semiellipses 9, 10, and 11 represents the direction of the line extending from the observer to the sun at whatever solar altitude the sun may be at the time the observation is made. The sizes of the vectorial diagrams in Fig. 22 represent only approximately the intensities of the diffusely transmitted light. It is difficult to draw these to scale because of the wide range of values encountered.

Since the luminance of the sun has decreased to about 1/32 of the "clear" value, the luminous density due to direct sunlight has decreased to about 3.1 foot-lumergs. The sky, however, is now very luminous, varying from 100,000 (or even 200,000) foot-lamberts near the sun to approximately 2000 foot-lamberts at a few degrees above the horizon. This high luminance, combined with the large solid angle which the sky subtends (2π steradians), increases the luminous density attributable to skylight to about 3 times that of the "clear" sky condition.

2. Medium haze.—As condensation increases, the luminance of the sky decreases. When the luminance of the sun (and the luminance density due to direct sunlight at the earth's surface) has diminished to about 0.1 percent of that for a "clear" atmosphere, the sun can be viewed directly without serious visual discomfort. The luminous density due to skylight has decreased appreciably but is still about twice as great as that due to the "clear" atmospheric condition. The sun is still visible; in some instances it appears as a sharp, clearly defined bright disk, and in others as a very bright spot, somewhat larger than the sun's disk, with poorly defined boundaries. This variation in appearance is probably due to the particle sizes of the condensed water vapor or to the spatial distribution of this condensation, or to combinations of the two. The sky near the sun may be described as white with a luminance much less than that of the sun's disk. This luminance decreases rapidly as the angular distance from the sun increases, merging finally into a light gray or dull white as the horizon is approached. This condition is designated as *medium haze*. The total luminous density is now about half of that for the "clear" condition and practically all of this (99.8 percent) is due to skylight. The atmospheric index A for this condition is 3.0. The loss of light is accounted for by the reflection outward into space at the upper surface of the cloud zone and by absorption within the cloud zone.

This condition is illustrated at 3 in Fig. 22. The intensity of the unscattered component is now appreciably less than for a clear atmosphere, as indicated by the width at C. The distribution curve of the diffused component is represented by the approximate semiellipse 9, of which the major axis is shorter and the minor axis longer than those for light haze, as illustrated at 8. As shown by the relative lengths of the vector lines, the sky luminance in the immediate vicinity of the sun is considerably greater than that at greater angular departures.

3. Heavy haze.—If condensation continues until the sun's disk is just visible or only a few times more luminous than the surrounding sky, which has now taken on a bright gray appearance in the vicinity of the sun, the condition which may be described as *heavy haze* is reached. Here, again, the sky in the immediate vicinity of the sun is appreciably more luminous and less gray in appearance than those parts farther away and nearer the horizon. As indicated at D, the intensity of the unscattered component has decreased almost to the vanishing point. The distribution of the scattered light is approximately as shown at 10, Fig. 22.

The luminance of the sun's disk and the luminous density due to direct sunlight have decreased to about 0.003 percent of the clear-atmosphere values. The cast shadow has practically disappeared, although there are of course *shaded* areas of lower luminance than those exposed to large solid angular expanses of the sky. The total luminous density made up almost entirely of the sky component, is about one-third of the "clear" value.

4. Light cloud.—When the thickness or density of the condensation has increased just a little beyond the *heavy haze* condition, the sun's disk becomes invisible. This condition is designated as *light cloud*.

If the cloud formation is of an approximately uniform character, the position of the sun may generally be recognized by a roughly circular bright area of rather large angular size. The luminance is greatest at the center of this luminous area (which corresponds approximately to the sun's position) and decreases rather gradually outward from this point in all directions. There are no cast shadows. The total luminous density is now a little more than one-fourth of that for the clear condition, all of this being attributable to sky light.

When the light cloud formation is not uniform but is striated, streaked, or mottled, the approximate position of the sun may, in many cases, be recognized by the presence of a bright spot of irregular shape or by a group of bright spots varying in shape and size clustered around the approximate position of the sun. The total luminous density is very nearly the same as that with the more uniform cloud condition mentioned in the previous paragraph. With these light cloud conditions the unscattered component of sunlight has disappeared, thus indicating that the sun's disk is no longer visible. The approximate semiellipse which represents the distribution of the scattered light (see 11 in Fig. 22) is approaching the form of a semicircle, although the major axis of the ellipse is still appreciably greater than the minor axis.

When the cloud formation is not uniform, and if the clouds are relatively low and moving more or less rapidly, the sun's disk may be visible intermittently and appear as described under *heavy haze*. This illustrates the very close relationship between the heavy haze and light cloud conditions. The word "haze" seems to have a rather definite connotation indicating an approximately uniform distribution (over relatively large solid angular extension) of the condensed water vapor. Thus, the most significant distinction between the *heavy haze* and the *light cloud* conditions may be based largely upon the *uniformity* or non-uniformity characteristics of the cloud formation.

The total luminous density, V_t, for light cloud differs so little from that for heavy haze that it seems inadvisable to assign different values of atmospheric index to these two conditions. For the purposes of photographic exposure, we may regard these two conditions as identical and assign a value of 5.0 as the atmospheric index for both, corresponding to a luminous density of one-third of that for a "clear" atmosphere.

5. Medium cloud.—If condensation proceeds further, the layer becomes denser or thicker, or possibly both, and the whole sky is filled with a dull gray cloud. There is no definite localization of the bright area indicative of the sun's position. The most luminous area of the sky is now at or near the zenith. This luminance decreases gradually from zenith to horizon, the zenith luminance being four or five times as great as that near the horizon. The luminous density at the earth's surface has decreased to about one-fifth of that for a "clear" atmosphere, the atmospheric index being 7.0. This condition is illustrated at 6 in Fig. 22, the distribution of the diffused light being illustrated at 12. Here, the semiellipse approaches more nearly to a semicircular form than for previous conditions. With a cloud formation of this density, about 75 percent of the light incident on the upper surface of the cloud zone is reflected back into space and some is absorbed in the cloud zone. The major axis of the semiellipse (at 12, also at 13, Fig. 22) now is coincident in direction with the straight line drawn from the observer to the zenith, rather than with the straight line from the observer to the sun as in the semiellipses shown at 8, 9, 10, and 11 in Fig. 22.

The atmospheric condition sometimes described by the phrase "just completely overcast" is closely associated with the medium cloud condition. It is descriptive of a rather uniform cloud, perhaps somewhat less dense or thick than the average *medium cloud*. The position of the sun is not definitely indicated by an area of maximum luminance. Maximum sky luminance is at or near the zenith, irrespective of the sun's zenith distance, and this decreases gradually in all directions as the horizon is approached. Possibly this condition should be placed about midway between the light and medium cloud formations as described in previous paragraphs, but for all practical photographic purposes it seems permissible to assign to the "just completely overcast" condition the same atmospheric index as that assigned to medium cloud, namely, 7.0. It is distinguished from the light cloud condition by the absence of an area of high luminance indicative of the sun's position, the highest luminance now being at or near the zenith.

6. Heavy cloud.—The thickness and density of the cloud layer may continue to increase beyond the medium cloud stage. The sky then takes on a dull gray appearance with relatively little decrease in luminance from zenith to horizon. There is no indication of the sun's position. As these heavier cloud conditions are encountered, it becomes increasingly difficult to recognize them with certainty by visual inspection. Since the human eye accommodates itself to very great ranges in absolute luminance levels, dependence must be placed very largely on the recognition of luminance differences, that is, by the contrast visible in the sky structure. The differentiation between medium and heavy cloud conditions must probably be based on the ratio of zenith to horizon luminance, since this ratio is sufficiently large to be recognized with fair certainty for the medium cloud condition, but appreciably lower and barely perceptible in the case of the heavy cloud. Actual measurements made on the sky luminance for the heavy cloud condition indicate a ratio of zenith to horizontal luminance of from 2.5 to 3.0. The luminance level and the luminance gradient are, however, so low that many observers fail to perceive the difference between zenith and horizon luminance, expressing the opinion that the luminance of the sky just above the horizon is not perceptibly less than that at the zenith. The total luminous density at the earth's surface has now decreased to about one-tenth of that for the "clear" atmosphere, thus giving an atmospheric index of 10. The distribution of the light scattered by the heavy cloud layer is illustrated by the curve 13 in Fig. 22 which approximates quite closely the semicircular form.

7. Dense cloud.—With still further increase in the amount of condensed water vapor in the atmosphere, a condition describable as *dense cloud* may be encountered. The sky then takes on a dark gray leaden appearance with very little if any perceptible change in luminance from zenith to horizon, even during midday hours, with the sun at an altitude of 50 degrees or more. The sensory or, perhaps we should say, the perceptual evaluation of the illuminance level is such as to suggest the approach of nightfall. While it is true that the human visual mechanism adapts itself to an extremely wide variation in luminance, the conditions existing with dense cloud formations are definitely perceived as depressed luminance levels. Perhaps the situation is best expressed by the word "gloomy." Luminous density under these conditions is about one-thirtieth of that for the "clear" condition and, hence, an atmospheric index of 15 must be assigned.

Under extreme conditions, clouds of even greater density may be encountered. Values of luminous density as low as 1.0 foot lumerg or less have been measured even for relatively high (greater than 50 degrees) solar altitudes. It seems unnecessary, however, to attempt to include these conditions in the construction of exposure guides since, when they are encountered, photographic operations become extremely difficult or impossible because the camera exposure must be increased 150- or even 300-fold as compared with that required under "clear" atmospheric conditions.

The diagram in Fig. 22 is intended to illustrate what happens to direct sunlight as it passes through the cloud zone. In addition to the direct sunlight there is, of course, incident upon the upper surface of the cloud zone a certain amount of light, skylight, which has been scattered by the molecules in that part of the atmosphere which rises above the cloud zone. It is generally conceded that for an atmosphere containing no condensed water vapor (or dust particles) the Rayleigh theory of molecular scattering is valid. Several attempts have been made to compute the sky luminance by the application of Rayleigh's theory. Among these may be mentioned those of King,[48] Hammad and Chapman,[49] Silberstein,[50] and Tousey and Hulburt.[51] Using Silberstein's treatment of this problem, computations have been made which indicate that when the sun is near the zenith, the maximum sky luminance is found at the horizon and is, on the average, about 3.5 times the minimum sky luminance which is found near the zenith.

Measurements of the water and dust content of the upper air have been made by the U. S. Weather Bureau[52, 53] and some recent measurements of sky luminance reported by Tousey and Hulburt[51] indicate that the atmosphere above the cloud zone does contain some scattering material (condensed water vapor and dust). The presence

Table XIV. Atmospheric index, A, for various atmospheric conditions.

Atmospheric condition	V_t	R_a	A
Clear	128	1.0	0
Light haze	91	1.4	15
Medium haze	64	2.0	3
Heavy haze	44	3.2	5
Light cloud	36		
Medium cloud	26	5	7
Heavy cloud	13	10	10
Dense cloud	4	32	15

of this above the cloud zone might tend to impart to the skylight incident on the upper surface of the cloud zone a somewhat more diffuse character than that deduced from the Rayleigh theory of molecular scattering. It appears very likely, however, that the diffuseness of this skylight resembles more closely that deduced from the Rayleigh theory than that obtained by actual measurement of sky luminance at the earth's surface under the "clear" atmospheric condition. In any case, however, it may be said without danger of serious error that the skylight incident on the upper surface of the cloud zone is multidirectional, coming from every point within the 2π steradians of solid angle subtended by the sky, and that its directional distribution approaches fairly closely to that of perfect diffusion.

This skylight, as it travels toward the earth, is scattered and diffused by passage through the cloud zone, although this scattering is not of such a nature as to change appreciably its distribution. Thus, if the skylight, after traversing the cloud zone, were added to the vectorial diagrams in Fig. 22, it would tend to make them somewhat more rounded and less elongated in form.

In Fig. 23 an attempt has been made to show graphically the appearance of the sun and the surrounding sky when four of the specific atmospheric conditions discussed in the previous paragraphs are encountered. These but poorly reproduce these appearances, but may render some assistance in visualizing these aspects of appearance which are associated with definite values of luminous density.

In Table XIII are given some numerical values which characterize approximately the various atmospheric conditions indicated in column 1. These apply specifically to the conditions which exist when the solar altitude is 50 degrees or more. The numbers in boldface type are derived from a large number of measurements, made in many cases by several observers, and are subject to only small uncertainties. Of the others, some are based on relatively few measurements, others are interpolated, and still others are estimated. However, we believe all the values in the table are sufficiently reliable for predicting correct camera exposures.

It will be noted that all the values given for the "clear" atmospheric condition are in boldface. This condition has been studied extensively, and hence more data are available for this condition than for any of the others. It is interesting to note that the luminance of the sun's disk (column 2) decreases very rapidly with the accumulation of condensed water vapor in the cloud zone. The luminous density, V_s, attributable to skylight, is relatively low for the *perfectly clear* condition, increasing to a maximum for the light haze condition, and then decreasing again continuously to the dense cloud condition. In column 9 are given values of relative camera exposure, R_a, and in column 10 the corresponding values of atmospheric index, A. The small figures placed beside some of the numbers indicate the approximate location in the sky to which these numerical values apply.

B. Summary of Clues for Clear, and Approximately Uniform Haze and Cloud Conditions

Clear.—High purity of the blue sky color. General low level of sky luminance. Cast shadows sharp, dark, and distinct.

Light Haze.—Sky color white, of high luminance, almost dazzling near the sun, producing a feeling of glare and visual discomfort. Cast shadows visible but more grayish than for clear condition.

Medium Haze.—Sky near sun white but not dazzling, general appearance—bright grayish white. Sun's disk may be viewed directly without serious visual discomfort. Cast shadows visible but faint, of low contrast and "soft" appearance.

Heavy Haze.—Sun's disk only a few times more luminous than the immediate surrounding sky. Sky a dull grayish white, cast shadows barely visible.

Light Cloud.—Sun's disk invisible or intermittently so. Sky, as a whole, light gray with maximum luminance in the immediate vicinity of the sun's position. No cast shadows.

Medium Cloud.—Sky, as a whole, dull gray with maximum luminance at the zenith, *not* near the sun's position which now cannot be determined by visual inspection. Sky luminance diminishes gradually in all directions from the zenith to about one-quarter of the zenith value near the horizon.

Heavy Cloud.—Sky dark gray with maximum luminance at the zenith but very little, if any, perceptual decrease of luminance toward the horizon. Position of sun indeterminate.

Dense Cloud.—Sky in general has a very dark gray, gloomy appearance with no perceptible luminance gradient from zenith to horizon. This condition produces conscious feeling of low luminance level usually associated with dusk and dawn conditions.

We shall now consider what happens when the atmospheric conditions described in the previous paragraphs remain constant, while the solar altitude decreases. Thus far we have assumed the altitude of the sun to be greater than 50 degrees and the air mass less than 1.3. Now, as the solar altitude decreases, the path length, which the sunlight incident upon the upper boundary of the cloud zone must traverse in passing through this zone, increases as shown in Fig. 21. This means that the appearance of the sun and sky (and the total luminous density at the earth's surface) will change as the solar altitude decreases without any change whatever in the amount of condensed water vapor in the cloud zone. Even if we could in some manner specify the amount of condensed water vapor, its particle size, distribution, etc., it still would not be a significant criterion of the diminution of the solar light by the cloud zone without taking into account solar altitude. We are, however, endeavoring to establish the relationship between the amount of light available at the earth's surface and the *appearance* of the sun and sky. For instance, consider the case of medium haze as evaluated by appearance when the sun is near the zenith (see Fig. 22 at 3). Let us assume that the amount of condensed water vapor in the cloud zone remains fixed, while the sun's altitude declines to approximately 30 degrees. The path length traversed by direct sunlight in passing through the cloud zone is doubled and the luminous density at the earth's surface attributable to direct sunlight is decreased to a greater extent than is accounted for by the increase of air mass ($m = 2.0$). The decrease in luminous density, V_d, because of the increase in air mass is only 20 percent, which is practically negligible for the purposes of predicting the required camera exposure. The effective thickness of the diffusing layer has, however, *doubled*, and this causes a large decrease in the total luminous density at the earth's surface. However, the appearance of the sun and sky has changed also and is approximately that illustrated at 5 in Fig. 22, which represents the light cloud condition and would be so appraised by visual inspection. The appearance of the sun and sky is therefore a significant criterion of the decrement in the solar light ascribable to the cloud layer regardless of solar altitude. With the sun at altitude 30 degrees and an atmospheric condition, *judged by visual inspection*, describable as *light cloud*, the total luminous density arriving at the earth's surface is given by the corresponding atmospheric index, A, which is 5.0, combined with the light index, L_v, for a solar altitude of 30 degrees, which is 1.0.

The fact that it may take a smaller amount of condensed water vapor to reduce the luminous density from its clear atmospheric value to some fraction thereof, when the sun is at a lower altitude than when it is near the zenith, is of no consequence. Hence, if atmospheric condition be defined in terms of the appearance of the sun and sky, and if these appearances can be identified definitely, then the diminution of the solar light by these atmospheric conditions can be estimated. *Appearance* cannot be defined in terms of a *single* characteristic of the sun and sky, because many factors contribute to the perceptual complex. Among these factors the most important are color, luminance, and contrast. Contrast depends upon chromaticity differences in addition to luminance differences. While the human visual system is usually not capable of appraising absolute luminance with certainty, it is quite capable of indicating general luminance levels.

There is some evidence, derived from our own experimental data, that luminous density values based upon the visual appraisal of the light, medium, and heavy cloud conditions, may in some cases be somewhat too high. These cases arise when the value of solar altitude is relatively low, less than 15 degrees. The error is not of high order, being approximately one or two units on the atmospheric index scale. Hence, it may be desirable under such conditions to increase the atmospheric index by one or two units, as compared with the atmospheric index indicated in Table XIV.

Non-Uniform Haze and Cloud Conditions.—The approximate uniform cloud condition assumed thus far is actually encountered quite frequently, but on many occasions it is necessary to deal with clouds which show marked striated and mottled non-uniformities and with a sky more or less filled with detached cloud masses varying greatly in shape, size, and position. Here, again, the appearance of the sun and of the sky immediately around the sun is the best criterion of available luminous density. Thus, if the sky is partially filled with cloud masses so broken that the sun is unobscured and the sky around it is of clear deep blue (clear atmosphere), and if the rift in the clouds is of sufficient size so that the entire scene to be photographed is flooded with sunlight of full strength, the luminous density is very nearly the same as for the clear atmospheric condition for the same solar altitude. Under such conditions the sun contributes the same luminous density as with a clear atmosphere. Some of the visible cloud surface illuminated by sunlight may be appreciably brighter, but others not receiving sunlight will be somewhat darker than the normal blue sky. On the average, the total contribution of sky to luminous density is about the same as that of the normal blue sky of a clear atmosphere. Since the best criterion of photographic exposure is minimum object luminance, if a part of the scene lies under the shadow of a cloud mass (even if the sun is unobscured at the camera position), it may be necessary to estimate the diminution of luminous density at that point as equivalent to light cloud, medium cloud, heavy cloud, or dense cloud. The darkness of this cloud shadow usually is a good indication of which atmospheric condition exists at that point.

When the sky is more or less filled with non-uniform or broken cloud masses and the particular part that covers the sun imparts to it and to the immediately surrounding sky areas the same appearance as that occurring with medium haze, then the luminous density is about the same as that for the *uniform* medium haze condition for the same solar altitude. This is true even if small or even fairly large areas of blue sky are visible, provided they are sufficiently removed from the sun's position so that direct sunlight of full "clear" atmospheric strength does not illuminate the scene. When considerable portions of the sky at appreciable angular distances from the sun are partially or wholly filled with cloud masses corresponding in general appearance to light or heavy clouds, the luminous density may be somewhat less than that for the uniform medium haze condition. Under these circumstances, it may be advisable to increase the atmospheric index for medium haze by one or two units, depending upon the darkness and extent of these cloud masses.

When the sun's disk is invisible and its position is not indicated by a definitely localized area of high luminance, the luminous density will be approximately the same as that assigned to the *uniform medium cloud* condition, provided that the non-uniform, broken, and irregular cloud masses which fill the major portion of the sky exhibit variations in luminance ranging from white or very light gray to medium gray. Here, again, there may be small areas of blue sky visible or small areas of darker gray, provided they do not make up a very large proportion of the total sky area.

The identification of the heavy and dense cloud conditions in the presence of marked non-uniformity must depend upon the observer's ability to estimate the average grayness of the non-uniform formation as compared with the grayness of the uniform heavy or dense cloud. In the case of non-uniform heavy cloud conditions, small areas of blue sky may be visible without seriously interfering with the general level of luminous density. The low luminous density associated with dense cloud formation seldom occurs without practically the entire sky being filled with dense, dark gray cloud masses, even though they may exhibit some variation in luminance from point to point.

We may summarize this discussion by emphasizing the fact that for the *clear, light haze, medium haze, dense haze*, or *light cloud* conditions, the amount of luminous density available at the observer's position on the earth's surface can be estimated by a careful appraisal of the sun and the sky in the immediate vicinity of the sun, say within an angular distance of 20 degrees. The identification of the medium, heavy, and dense cloud conditions, however, requires an estimate of the average luminance of the entire sky. This estimation is assisted to a considerable extent by the luminance contrast. With non-uniform skies, the structure of the *light* and *medium* cloud formation usually exhibits appreciably greater variations in luminance from point to point than with the non-uniform heavy and dense formations.

The indices for the atmospheric conditions which we have attempted to describe are shown in Table XIV.

In this installment, various methods for evaluating the amount of light *arriving* at the earth's surface from the sun and the sky have been discussed. From this analysis it has been concluded that the most significant measure of the amount of light, for the purpose of predicting correct camera exposure, is *luminous density* (volume density of luminous energy) expressed as lumergs per cubic foot. The dependence of luminous density, unmodified by the presence of adjacent terrestrial objects, upon *solar altitude* and *atmospheric condition* has been established.

Before this information can be used for the computation of correct camera exposure, it will be necessary to establish the relationship between the *arriving* luminous energy and the minimum luminance found in scenes of various types. Under certain circumstances, it is also necessary to take into account the relation between the direction of the optical axis of the camera system and the direction from which the predominant illuminance enters the scene space. A third factor which is of some importance is the average photographic efficiency of the luminous energy which is reflected by the scene element of minimum luminance.

In the following and final installment of this communication, the factors mentioned in the previous paragraph will be discussed in detail and numerical evaluations established. Up to this point, the treatment of the subject has been one of rather minute analysis. To make this information conveniently useful to the photographer, it will be necessary to reverse the process and, as it were, to integrate and simplify by allowable approximations so that camera exposure can be computed easily and quickly by use of slide-rule types of computers or by appropriately organized nomographs.

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The Reduction of Apparent Contrast by the Atmosphere

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A veil of atmospheric haze reduces the visibility of all distant objects by decreasing their apparent contrast. In this paper equations are derived which describe the manner in which the apparent contrast of any object depends upon the distance of the observer. The treatment is not limited to horizontal paths of sight, but applies also to the apparent contrast of objects on the ground as seen from the air, and to the apparent contrast of objects aloft as viewed from the ground. The equations are not limited to the case of a homogeneous standard atmosphere; they may be applied to many kinds of non-standard atmospheric conditions. For every path of sight there exists a luminance level which will be transmitted unchanged. The apparent luminance of any receding object approaches this equilibrium level. For many paths of sight the equilibrium luminance is matched by the luminance of some portion of the horizon sky.

THIS paper is an outgrowth of research begun during the war as part of a study of the visibility of distant objects;[1] it is presented in the belief that the theory may find peacetime uses. The limiting range at which a given object will be visible can be predicted[2] from data concerning contrast thresholds for the human eye[3] if a proper allowance is made for the reduction in apparent contrast caused by the atmosphere.[4,5] It is the purpose of this paper to set forth the laws which govern such a reduction of contrast along paths of sight which may be horizontal or may be inclined upward or downward. The simplest case is represented by an atmosphere uniform in lighting and in composition but decreasing regularly in density with increasing altitude. Such an idealized atmosphere will be treated first, but a method for allowing for departures from the idealized conditions will be presented near the end of the paper.

1. FUNDAMENTALS

The apparent luminance of any distant object is governed by two processes which operate concurrently: (1) light emanating from the object is gradually attenuated by scattering and by absorption; (2) daylight is scattered toward the observer all along the path of sight. The balance between that portion of the light emanating from the object which reaches the observer and the space light contributed by the intervening air determines the apparent luminance of the object. For example, the apparent luminance of a dark mountain increases with distance because of the addition of space light until the mountain finally is invisible because its apparent luminance matches that of the sky background. In like manner, the apparent luminance of a sunlit, snow-clad peak decreases with distance until, at a sufficient range, it too matches its sky background. In this case the reduction in apparent luminance is due to attenuation of light from the mountain by scattering and by absorption.

An insight into the relation between these two processes can be gained by regarding the atmosphere as a diffusing material, lighted throughout by natural sources such as the sun or moon, the sky, and the ground.** A useful method for describing the behavior of diffusing materials wherein both primary and secondary scattering play part was proposed by Schuster[6] in 1903, and it has subsequently been employed to describe the optical properties of most of the diffusing materials of interest to industry.[7] In an earlier paper,[8] the Schuster-type treatments by

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** An approach to the problem by the integral equation method has been made by E. S. Kousnetsov, whose paper entitled "Theory of non-horizontal visibility" appears in the Bulletin de L'Academe des Sciences de l'Union des Republiques Sovietiques Socialistes, No. 5 (1943).
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various authors were classified in accordance with the number of constants which they employed to characterize the particular diffusing material with which they were concerned. In the case of the atmosphere it has been found that two specially defined constants suffice under most of the commonly encountered seeing conditions.

1.1 The Differential Equations

Schuster conceived of two fluxes moving in opposite directions through the diffusing material, each contributing back-scattered flux to the other. Simultaneous differential equations can be used to describe the changes in these fluxes as they pass through a lamina of path. In Fig. 1 an extended emitting object has a spectral radiant emittance, W_0, in the direction of the observer, who is assumed to view the surface of the object normally along a path of sight inclined at an angle θ with the horizontal. At a distance r is a parallel-sided flat lamina of atmosphere having a thickness dr. The surfaces of the lamina are perpendicular to the line of sight. Let the diffused spectral irradiance on the lower side of the lamina be denoted by t, and the diffused spectral irradiance on the upper side of the lamina be denoted by s.

In passing through the lamina, the upward-moving radiant flux, t, will be diminished by absorption and by scattering, the magnitude of the change (dt/dr) being proportional to the flux t. Let the constant of proportionality be written $\mu_r + B_r$, where μ_r will be called the absorption coefficient and B_r the back-scattering coefficient, respectively. The subscript $_r$ denotes that, in general, these quantities vary along the path

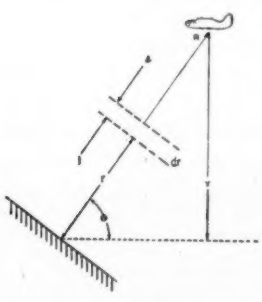

FIG. 1.

of sight. The flux t will also be augmented within the lamina by space light (l_t), and by back-scattered light from the oppositely moving flux body $(B_r s)$. In addition to its diffuse emittance, the object may also produce a directed beam. The diffused flux t will then be augmented by scattering from the beam $(F_r' I_r')$, where I_r' annotes the spectral irradiance on the lamina caused by the directed beam, and the proportionality coefficient F_r' is the forward-scattering coefficient for directed flux. The primes are used to indicate that these quantities refer to light which is not diffused. The notation adopted here conforms with the recommendations of the International Commission on Illumination for the specification of opal glass.[9] All of the above-mentioned changes in t are concurrent, as indicated by the following differential equation:

$$dt/dr = -\mu_r t - B_r t + l_t + B_r s + F_r' I_r'. \qquad (1)$$

Corresponding concurrent changes take place in the downward moving diffused flux s when it traverses the lamina. These changes are represented by the following differential equation wherein the proportionality coefficient B_r' is the backward-scattering coefficient for directed flux.

$$-ds/dr = -\mu_r s - B_r s + l_s + B_r t + B_r' I_r'. \qquad (2)$$

Differential equations (1) and (2) may be thought of as a steady-state equation of continuity for flux permeating the atmosphere. They involve the assumption that the radiation is monochromatic and the assumption that the medium may be divided laminarwise without destroying its over-all homogeneity. The treatment is thermodynamic in nature, and attempts no description of the behavior of light in passing over the airborne particles. For this reason, effects such as those discussed by Rayleigh will appear as spectral variations in the absorption and scattering coefficients.

The contributions arising from space light are some fractions of the radiant density in the lamina. Therefore, the terms l_t and l_s can be replaced by $\tau_r q_r$ and $\sigma_r q_r$, where q_r is the spectral radiant density of space light at the lamina and τ_r and σ_r are *scattering-rate coefficients*.

If the observer's path of sight is horizontal,

[9] International Commission on Illumination, Neuvième Session, p. 3 (1937).

and if the atmosphere is homogeneous in its lighting and in its composition, the spectral radiant density q is the same at all points. Along inclined paths of sight, however, a variation of radiant density with altitude might be expected. Experiments*** conducted during the recent war by the Louis Comfort Tiffany Foundation, under a contract with the Office of Scientific Research and Development, sought to measure this variation, but none was discovered up to the highest altitude tested, 18,000 feet. In all cases the experiments were conducted with solar elevation in the neighborhood of 50 degrees. There seems, however, reason to expect that variations of q with altitude would have been found for low values of solar elevation. For example, just after sunset the radiant density is much greater at high altitudes than it is near the ground. The range of solar elevation within which the variation of q with altitude may be neglected is not known. A further experimental investigation of this matter should be undertaken. In the discussion which follows, q will be assumed to the same at all points along the path of sight.

The terms $B_r s$ and $B_r t$ which represent the contributions arising from back-scattering from the oppositely moving flux body are, to a first approximation, constant along the path of sight, except in the case of objects whose inherent spectral radiance is much greater than that of their background. If $B_r s$ and $B_r t$ are constant, they may be regarded as equivalent to an increase in the radiant density q. Hence, in Eq. (1) the terms $l_t + B_r s$ can be replaced by $\tau_r q$, and in Eq. (2) the terms $l_s + B_r t$ by $\sigma_r q$.

Inasmuch as the flux is attenuated both by absorption and by scattering, the terms $\mu_r t + B_r t$ in Eq. (1) can be replaced by $\beta_r t$, and the terms $\mu_r s + B_r s$ in Eq. (2) can be replaced by $\beta_r s$, where β_r is referred to as the *atmospheric attenuation coefficient*.

In the interest of simplicity let the discussion be restricted to cases in which the terms $F_r' I_r'$ and $B_r' I_r'$ are negligible; the special case of an intense, highly collimated source, such as a searchlight, being treated separately in Section 2.4 of this paper. As a further simplification, let the discussion be limited to those cases in which the effect of the object upon q can be neglected throughout all but a negligible portion of the path of sight; by means of this restriction, the 1-dimensional differential equations (1) and (2) can be applied to situations in which the object subtends but a small solid angle at the eye of the observer. The discussion of the reduction by the atmosphere of the apparent contrast of objects which subtend a large angle at the eye of the observer will be reserved for a subsequent paper. Equations (1) and (2) may now be written

$$dt/dr = -\beta_r t + \tau_r q, \qquad (3)$$

$$-ds/dr = -\beta_r s + \sigma_r q. \qquad (4)$$

Because of the assumptions made in passing from Eqs. (1) and (2) to Eqs. (3) and (4), the latter are not intended to be valid for objects possessing inherent radiance greatly in excess of their background, or along steeply inclined paths of sight when the sun is low. However, Eqs. (3) and (4) will be discussed fully because they treat correctly most of the commonly encountered problems in visibility, and because many of the principles of atmospheric optics can be derived simply from them. A more general discussion based upon Eqs. (1) and (2) will be reserved for a subsequent paper.

1.2 The Solutions

Since $B_r s$ and $B_r t$ have been assumed constant along the path of sight, the feedback of flux is no longer a variable, and Eqs. (3) and (4) need not be solved simultaneously. They may be rearranged in a form permitting integration by replacing β_r by $\beta_0 f(r)$, τ_r by $\tau_0 f(r)$, and σ_r by $\sigma_0 f(r)$, where β_0, τ_0, and σ_0 are the values assumed by the respective variables at the lower end of the path of sight. Equations (3) and (4) may now be written:

$$\int_{W_o}^{W_R} \frac{dt}{-\beta_o t + \tau_o q} = \int_o^R f(r)dr, \qquad (5)$$

$$\int_{W_o}^{W_R} \frac{ds}{-\beta_o s + \sigma_o q} = -\int_R^o f(r)dr. \qquad (6)$$

The limits of integration in Eq. (5) are arranged to correspond with the case of an observer looking

*** The experiments referred to in this paper were conducted under the author's supervision by the Louis Comfort Tiffany Foundation under contract OEMsr 597 with the Office of Scientific Research and Development.

downward along a slant path of length R at an object whose ***apparent spectral radiant emittance*** is W_R, but whose ***inherent spectral radiant emittance*** is W_0. The limits of Eq. (6) are arranged to represent the corresponding case of an observer looking upward along the slant path. It will be noted that the equations are identical except for the scattering-rate coefficients τ_0 and σ_0.

In the case of a horizontal path of sight through a homogeneous atmosphere, $f(r)=1$ since β_0, τ_0, and σ_0 are the same at all points. The integrals on the right then reduce to R, the range of the object. In the case of an inclined path of sight, $f(r)\neq 1$ because the air density decreases with increasing altitude (see Section 3.1). Let the integral of $f(r)$ along the path of sight be called the ***optical slant range*** and denoted by $\bar{R}$. Thus

$$\bar{R}=\int_0^R f(r)dr. \tag{7}$$

Physically, $\bar{R}$ represents that horizontal distance in a homogeneous atmosphere for which the attenuation is the same as that actually encountered along the true path of length R.†

The integrated forms of Eqs. (5) and (6) may now be written:

$$W_R=\frac{\tau_o q}{\beta_o}(1-e^{-\beta_o\bar{R}})+W_o e^{-\beta_o\bar{R}}, \tag{8}$$

$$W_R=\frac{\sigma_o q}{\beta_o}(1-e^{-\beta_o\bar{R}})+W_o e^{-\beta_o\bar{R}}. \tag{9}$$

Equation (8) applies to an observer looking downward along a slant path; Eq. (9) to an observer looking upward. Both equations are seen to be comprised of two terms, the first representing the space light and the second representing flux from the object which reaches the observer.

If the quantities in Eqs. (8) and (9) showed no wave-length dependence, these relations would hold for the apparent luminance of distant objects. The field experiments described in Section 2.4 indicate that for achromatic objects the attenuation of luminance follows the same law. Hence, for practical purposes, Eqs. (8) and (9) can be written in terms of the ***inherent luminance*** of the object B_0 and its ***apparent luminance*** B_R as seen from a distance R:

$$B_R=\frac{\tau_o q}{\beta_o}(1-e^{-\beta_o\bar{R}})+B_o e^{-\beta_o\bar{R}}, \tag{10}$$

$$B_R=\frac{\sigma_o q}{\beta_o}(1-e^{-\beta_o\bar{R}})+B_o e^{-\beta_o\bar{R}}, \tag{11}$$

where the symbols σ_0, τ_0, q, and β_0 now refer to luminous quantities.

An equation of this type for horizontal paths of sight was published in 1924 by Koschmieder.[12] His derivation is given by Middleton,[13] whose standard text on visibility includes an extensive survey of the literature.

2. VISIBILITY

From the standpoint of visibility the most important consequence of Eqs. (10) and (11) is the result that luminance differences are exponentially attenuated:

$$(\mathbf{B}_R-B_R)=(\mathbf{B}_o-B_o)e^{-\beta_o\bar{R}}, \tag{12}$$

where **B** refers to the luminance of the object and B refers to the luminance of the background. Since the perceptual capacity of the human eye is best described in terms of contrast thresholds, a relation similar to Eq. (12) but involving the contrast of the object against its background is required. Let the ***inherent contrast*** of the object C_0 and the ***apparent contrast*** C_R be defined as follows:

$$C_o=\frac{\mathbf{B}_o-B_o}{B_o}, \tag{13}$$

$$C_R=\frac{\mathbf{B}_R-B_R}{B_R}. \tag{14}$$

† An analogous procedure was adopted in a treatment of this subject by the Eastman Kodak Company in a monograph on aerial haze published in 1923 (reference 10), and more recently in a paper by Tousey and Hulburt of the Naval Research Laboratory (reference 11).

[10] "Aerial haze and its effect on photography from the air," Research Laboratory, Eastman Kodak Company, D. Van Nostrand Company, Inc. (1923).

[11] R. Tousey and E. O. Hulburt, J. Opt. Soc. Am. **37**, 78 (1947).

[12] H. Koschmieder, Beitr. z. Phys. d. freien Atm. **12**, 33 (1924), and **12**, 171 (1924).

[13] W. E. K. Middleton, *Visibility in Meteorology* (University of Toronto Press, Toronto, 1941).

Then Eq. (12) may be written

$$C_R = \frac{B_o}{B_R} C_o e^{-\beta_o \bar{R}}. \qquad (15)$$

Equation (15) is the law of contrast reduction by the atmosphere expressed in its most general form.

2.1 Visibility Upward

In the case of an observer looking upward, B_R in Eq. (15) is the apparent luminance of the sky in the direction of the object, while B_o is the apparent luminance of the sky in the direction of sight as seen by an observer at the object. From Eq. (11) the luminance of the sky as seen from the ground is

$$B_R = \frac{\sigma_o q}{\beta_o}(1 - e^{-\beta_o \bar{R}_{0,\infty}}), \qquad (16)$$

where subscripts $0, \infty$ indicate that Eq. (7) is to be integrated from zero to infinity. Similarly,

$$B_o = \frac{\sigma_o q}{\beta_o}(1 - e^{-\beta_o \bar{R}_{R,\infty}}). \qquad (17)$$

Substituting (16) and (17) in (15) yields the law of atmospheric contrast reduction for an observer looking upward along an inclined path of sight:

$$C_R = C_o e^{-\beta_o \bar{R}} \left[\frac{1 - e^{-\beta_o \bar{R}_{R,\infty}}}{1 - e^{-\beta_o \bar{R}_{0,\infty}}} \right]. \qquad (18)$$

The factor $\sigma_0 q/\beta_0$ has vanished, leaving the contrast attenuation affected only by the nondirectional quantity β_0, and $\bar{R}$ which, in a homogeneous atmosphere, depends only on the inclination θ of the path of sight and R. Thus (see Section 3.1) the contrast attenuation for an observer looking upward along an inclined path is independent of the direction of the sun. For example, an object of fixed inherent contrast moving at constant altitude in a circular path centered on the observer will present the same apparent contrast at all points of its path. For the special case of a horizontal path of sight, this theorem is equivalent to the common statement that the daylight visual range (see Section 2.4) is the same in all directions. When objects are viewed against the sky along horizontal paths of sight, Eqs. (15) and (18) reduce to

$$C_R = C_0 e^{-\beta_0 R}. \qquad (19)$$

2.2 Optical Equilibrium

Equation (3) shows that $dt/dr = 0$ when $t = \tau_r q/\beta_r = \tau_0 q/\beta_0$. Similarly, Eq. (4) shows that $ds/dt = 0$ when $s = \sigma_r q/\beta_r = \sigma_0 q/\beta_0$. Thus certain luminance levels are transmitted by the atmosphere without attenuation. These equilibrium levels are such that within each lamina of path the added space light exactly compensates for the attenuation. This condition has been called *optical equilibrium*.

It is a matter of common experience that the apparent luminance of the horizon does not change when an observer moves toward it. This indicates that in a homogeneous atmosphere, wherein q, σ_0, τ_0, and β_0 do not vary along the path of sight, the luminance of the horizon is the equilibrium value. Since the luminance of the horizon depends upon direction relative to the sun, and since β_0 and q do not vary with direction, there must be a compensating directional variation in the scattering-rate coefficients τ_0 or σ_0.

It often happens that some horizontal path of sight bears the same angle with respect to the sun as does the inclined path of sight. This is illustrated by Fig. 2. Usually there are two horizontal directions, n and n', from which sunlight is scattered at the same angle as the sunlight which is scattered downward along the slant path, and two opposite directions, m and m', from which sunlight is scattered at the same angle as the sunlight which is scattered upward along the slant path. Thus the value of $\tau_0 q/\beta_0$ for the slant path equals the luminance of the horizon sky B_m as seen by an observer at the object looking in the m or m' direction. Similarly, B_n, the luminance of the horizon sky in the n or n' directions, equals $\sigma_0 q/\beta_0$.

As a consequence of the phenomenon of optical equilibrium an observer looking upward sees the apparent luminance of any receding object approach the luminance of the horizon sky in the n directions. As seen by an observer looking horizontally, the apparent luminance of a receding

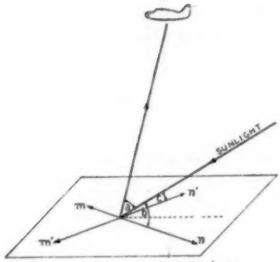

FIG. 2. Arrows n and n' indicate the directions in which the luminance of the horizon sky is determined by light scattered from the rays of the sun at the same angle as light scattered downward along the path of sight. Arrows m and m' indicate the directions in which the luminance of the horizon sky is determined by light scattered from the rays of the sun at the same angle as light scattered upward along the path of sight.

object changes with distance in such a manner as to approach the luminance of the horizon sky in the direction of the object. An ascending observer sees the apparent luminance of all objects on the ground approach the apparent luminance seen by an observer on the ground who looks at the horizon sky in the m directions.

2.3 Visibility Downward

Aviators commonly report a "circle of visibility" beyond which nothing can be distinguished on the ground. In a sense this is a misnomer, because the curve often departs markedly from circular shape. An insight into this phenomena can be gained by combining Eqs. (10) and (15) as follows:

$$C_R = C_o\left[1 - \frac{\tau_0 q}{\beta_o B_o}(1 - e^{\beta_o \bar{R}})\right]^{-1}. \qquad (20)$$

When the relation between the direction of the sun and the path of sight is such that the m and n horizontal directions exist (see Fig. 2), $B_m = \tau_0 q/\beta_0$ and the law of contrast attenuation

TABLE I.

Sky condition	Ground condition	Sky-ground ratio
Overcast	Fresh snow	1
Overcast	Desert	7
Overcast	Forest	25
Clear	Fresh snow	0.2
Clear	Desert	1.4
Clear	Forest	5

may be written:

$$C_R = C_o\left[1 - \frac{B_m}{B_o}(1 - e^{\beta_o \bar{R}})\right]^{-1}. \qquad (21)$$

The quantity B_m/B_0 has been called the *sky-ground ratio*. Typical values of this quantity are given in Table I. Both B_m and B_0 can be measured with a conventional luminance photometer, but the evaluation of $\tau_0 q/\beta_0$ by three separate photometric measurements requires uncommon apparatus. Such measurements can be made, however, when because of geometry the m and m' directions do not exist, or when obstacles or non-uniform weather conditions make direct measurement of B_m impossible or meaningless. Henceforth, the term *sky-ground ratio* will be used to denote $\tau_0 q/\beta_0 B_0$ under all circumstances.

The shape of the "circle of visibility" cannot be expressed analytically, even for an idealized homogeneous atmosphere, partly because the inherent contrast of natural terrains usually depends upon the bearing of the sun relative to the line of sight, and partly because the magnitude of the just-visible apparent contrast depends upon both the angular size of the object and the state of adaptation of the observer's eye. A nomographic method for taking account of these variables has been described,[2] but that treatment cannot yield an analytic expression for the "circle of visibility." Under circumstances such that the "circle" is defined by the condition that the ratio C_R/C_0 is constant, the optical slant range $\bar{R}$ is seen to depend upon τ_0, q, β_0, and B_0. Of these, only τ_0 depends upon the direction of the line of sight, and hence it might at first be supposed that the "circle of visibility" is a functional polar plot of τ_0 or of the brightness B_m of the horizon sky as seen by an observer on the ground. This is not strictly true, however, because the relation between $\bar{R}$ and R depends upon the angular elevation θ of the path of sight.

2.4 Horizontal Visibility

When an object is viewed along a horizontal path of sight, $\tau_0 q/\beta_0 = B_H$, the luminance of the horizon sky in the direction of the object. The law of contrast attenuation then becomes

$$C_R = C_0\left[1 - \frac{B_H}{B_0}(1 - e^{\beta_0 R})\right]^{-1}. \qquad (22)$$

In the case of an object viewed along a horizontal path of sight against the horizon sky as a background, $B_0 = B_H$ and Eq. (22) reduces to the form of Eq. (19).

An experiment designed to test the validity of Eq. (19) for dark targets was conducted by the Louis Comfort Tiffany Foundation under their OSRD contract. Large, glossy black, vertical panels were erected at distances of approximately 1500, 3000, and 6000 yards, and their apparent luminances and that of the horizon sky were measured by means of both photographic and photoelectric telephotometers. Figures 3 and 4 show the results of two such tests.

In discussing Eqs. (8) and (9) it was pointed out that the second term on the right represents the fraction of the flux emanating from the object which reaches the observer. Let the *contrast transmittance* T_R of a horizontal path of length R be defined from Eq. (19) as

$$T_R = C_R / C_o = e^{-\beta_0 R}. \qquad (23)$$

The contrast transmittance T of a unit distance (mile, yard, etc.) of atmosphere is

$$T = e^{-\beta_0}. \qquad (24)$$

Equation (24) can, if desired, be combined with Eqs. (8) through (22). For example, Eqs. (10) and (19) may be written:

$$B_R = B_H(1 - T^{\bar{R}}) - B_o T^{\bar{R}}, \qquad (25)$$

$$C_R = C_o T^R. \qquad (26)$$

When the light emitted by the object is intense and highly collimated, Eqs. (3) and (4) and relations derived therefrom should not be used directly, for the terms of Eqs. (1) and (2) which involve I_r' must be considered. Although a completely general solution for the searchlight problem appears very involved, advantage may be taken of the limited angular opening of the searchlight beam and the problem may be divided into two independent limiting cases. Consider first the unlighted searchlight as an object. Its inherent luminance then differs from that of its background by no more than does that of other non-self-luminous objects, and therefore its apparent luminance is given by Eqs. (8) through (22). When the searchlight is lit, a highly collimated narrow beam is superimposed upon the path of sight, causing an irradiance I_R' at the observer. Because the collimated beam is narrow, the secondary scattering terms are negligible,[14] and the differential equation for the collimated beam is

$$dI_r'/dr = -\mu_r' I_r' - B_r' I_r' - F_r' I_r' = -\beta_o' f(r) I_r', \qquad (27)$$

where $\beta_0' = \mu_0' + B_0' + F_0'$.

From the solution of Eq. (27), the transmittance of the atmosphere for a collimated beam (the *beam transmittance*) is

$$T_R' = e^{-\beta_0' \bar{R}}. \qquad (28)$$

A photoelectric *transmissometer* for measuring T_R' has been developed by members of the staff of the National Bureau of Standards.[15] Comparison of β_0 as computed from the slope of the lines in Figs. 3 and 4 with corresponding values of β_0' computed by means of Eq. (28) from data secured with a transmissometer show agreement to within 10 percent. Although this agreement suggests that measured values of T_R' may sometimes be a useful substitute for data on T_R, differences should be expected, and an extensive

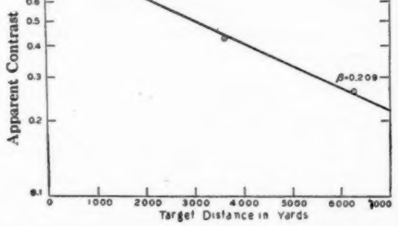

FIG. 3. Apparent contrast of distant black panels as measured with a photographic telephotometer. From the slope of the line: $\beta_o = 0.209$ per thousand yards; $v = 18{,}700$ yards

Time: 11 A.M.
Sky: 7/10 cirrus H_6
Ceiling: 25,000 feet
Estimated visibility: between 3 and 12 miles
Atmospheric pressure: 1011.9 millibars
Temperature: 74 degrees F
Dew point: 65 degrees F
Relative humidity: 74 percent
Wind: NW light.

[14] F. Benford, J. Opt. Soc. Am. **36**, 524 (1946).

[15] C. A. Douglas and L. L. Young, Technical Development Report No. 47, Civil Aeronautics Administration.

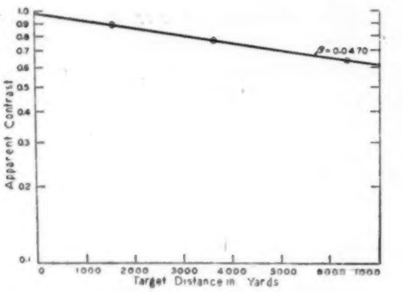

FIG. 4. Apparent contrast of distant black panels as measured with a photographic telephotometer. From the slope of the line: $\beta_o = 0.047$ per thousand yards: $v = 83{,}200$ yards

Time: noon
Sky: clear
Estimated visibility: 50 miles
Atmospheric pressure: 1014.6 millibars
Temperature: 57 degrees F
Dew point: 47 degrees F
Relative humidity: 69 percent
Wind: NW 6 miles per hour.

experimental comparison of these two quantities will be required before the circumstances under which data on T_R' can be used in visibility problems will be known.

The most commonly used quantities for expressing the attenuating properties of the atmosphere are β_0, T, and the *meteorological range* v. Meteorological range is defined as that horizontal distance for which the contrast transmittance of the atmosphere is two percent. From Eqs. (23) and (24), v is seen to be related to β_0 and T by the relation††

$$v = \frac{3.912}{\beta_o} = \frac{1.699}{\log_{10}\frac{1}{T}}. \tag{29}$$

Meteorologists customarily make a visual judgment of the optical effect of the atmosphere by inspecting distant black objects seen against the horizon sky. By international agreement[13,16] the *daylight visual range* is the distance at which a large dark object on the horizon is just recognizable. By a "large dark object" is meant an object that subtends so large an angle at the eye of the observer that if the subtended angle were greater the reported value of daylight visual range would not be changed. Because the distant dark objects available to meteorologists are rarely of sufficient angular size to fulfill this requirement, the reported "*visibility*" is usually somewhat less than the true daylight visual range. A comparison between "visibility" as reported by the staff meteorologist of the Tiffany Foundation and meteorological range as calculated from measurements of beam transmittance showed that, on the average, the reported "visibility" was three-fourths of the meteorological range. This conclusion is in agreement with the results of a similar experiment conducted before the war by personnel of the National Bureau of Standards.[15]

2.5 Allegedly Complicating Effects

Two allegedly complicating effects are referred to in the literature[13] as the *ground-glass plate effect* and the *edge effect*. The former refers to loss of sharp detail by low angle scattering, and the latter refers to an addition to the apparent luminance of an object because of light diffused around its edges. Since neither effect appeared to be supported by data of certain validity or by sound theoretical reasoning, the Louis Comfort Tiffany Foundation undertook, under its OSRD contract, to search for the effects and to determine their magnitudes under typical conditions.

The ground-glass plate effect was explored photographically. Distant resolving power targets were photographed in both clear and hazy weather with a camera having a focal length of 10 feet. When the contrast (gamma) of the photographic process was made equal to $1/T_R'$ determined from measurements of beam transmittance, the targets were resolved equally well in all photographs. The experiment was repeated using natural objects, and the conclusion was reached that no fine details were obliterated by haze. This conclusion concurs with the theoretical prediction of Middleton.[17]

The edge effect was explored with a high precision photoelectric telephotometer. This instrument was used to compare the apparent lumi-

†† It will be noted from Eqs. (24) and (29) that $\beta_o = 2.303D$, where D is the optical density of a horizontal path of unit length.

[16] Organization Météorologique Internationale, "Conférence des directeurs à Varsovie 1935," Vol. 1, No. 29 Leyden 1936.

[17] W. E. K. Middleton, J. Opt. Soc. Am. **32**, 139 (1942).

TABLE II.

Altitude in feet	Relative number of molecules per unit volume
0	1.000
1,000	0.956
2,000	0.918
3,000	0.878
4,000	0.841
5,000	0.804
6,000	0.770
7,000	0.736
8,000	0.703
9,000	0.672
10,000	0.642
12,000	0.586
14,000	0.534
16,000	0.485
18,000	0.440
20,000	0.399
22,000	0.361
24,000	0.326
26,000	0.295
28,000	0.266
30,000	0.239

nance of several equidistant black objects visible against a background of horizon sky. The angular size of the objects ranged from 0.8 minute to more than one degree. No difference in the apparent luminance of the objects was found.

3. OPTICAL SLANT RANGE

Throughout the foregoing discussion the optical slant range $\bar{R}$ has been left as a completely general function of the actual slant range R (see Eq. (7)). All the equations involving $\bar{R}$ thus far derived are, therefore, valid regardless of the nature of the variation of the coefficients β_r, σ_r, and τ_r with altitude. However, means for evaluating $\bar{R}$ must be provided before most of the preceding equations become useful.

3.1 The Optical Standard Atmosphere

Meteorologists sometimes refer to a "standard atmosphere" in which pressure varies with altitude in a prescribed manner and in which there is a constant lapse rate.[18] Let an *optical standard atmosphere* be defined as one in which the scattering particles are the same at all altitudes, but in which the relative number of particles per unit of volume (N) decreases regularly with altitude in the manner shown in Table II. The figures in Table II have been obtained by applying the equation of state of a perfect gas to the "standard atmosphere."

An analytic expression for the data in Table II, would permit the integration indicated by Eq. (7) to be performed. Figure 5 shows a semilogarithmic plot of N as a function of altitude. The points represent the optical standard atmosphere as defined by Table II. The straight line has the equation

$$N/N_o = e^{-y/21,700}, \tag{30}$$

where y is the altitude above sea level expressed in feet. Along any slant path $y = r\,\sin\theta$, and thus, if $f(r)$ is assumed equal to N/N_o, Eq. (7) becomes

$$\bar{R} = 21,700 \csc\theta\left[e^{-R_1 \sin\theta/21,700} - e^{-R_2 \sin\theta/21,700}\right]. \tag{31}$$

In the special case of an observer at sea level looking upward, or of an observer aloft looking at an object at sea level, Eq. (31) becomes

$$\bar{R} = 21,700 \csc\theta\left[1 - e^{-R \sin\theta/21,700}\right]. \tag{32}$$

In Eqs. (31) and (32) all distances are to be expressed in feet.

Figure 6 shows a plot of Eq. (32) for values of R from zero to 425,000 feet. The solid lines correspond with various values of θ from zero to 90 degrees. The broken curves indicate loci of equal altitude. Such a plot has been called an *optical slant-range diagram*. Figure 7 is an expansion of Fig. 6 near the origin.

3.2 Non-Standard Conditions

Non-standard atmospheric conditions are of common occurrence; often the path of sight

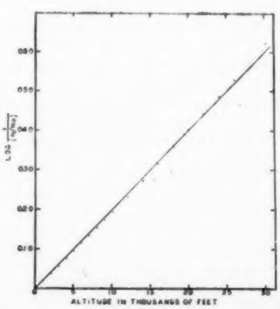

FIG. 5. The points represent the relative number of scattering particles per unit of volume in an optical standard atmosphere.

[18] *Encyclopaedia Britannica*, **3**, 129 (1945).

traverses strata, at the boundaries of which the optical properties of the atmosphere may change abruptly. Between stratum boundaries, however, the coefficients may vary in the manner of the optical standard atmosphere. This type of departure from standard conditions is readily allowed for on the optical slant-range diagram by a procedure illustrated in the following example.

Suppose that on a clear summer day the layer of air near the ground has the typical murky appearance caused by large suspended particles of water. Such ground haze often has a sharply defined upper boundary, above which the atmosphere contains very little condensed water. Above such a boundary the meteorological range is usually several times as great as below it. Figure 8 illustrates how the discontinuity can be represented on the optical slant-range diagram. For simplicity, only the curve for $\theta = 25$ degrees is shown. Let it be assumed that the boundary occurs at an altitude of 5000 feet, and that the meteorological range is five times greater above the boundary than below it. Beginning at the point corresponding with 5000 feet, a new curve has been drawn having five times the slope of the original curve. The relation between $\bar{R}$ and R is then represented by the accentuated line which follows the normal curve up to altitude 5000 feet and the steeper curve thereafter. If the boundary is diffuse rather than sharp, the accentuated line can be rounded off to avoid the abrupt change in slope.

The character and altitude of stratum boundaries can be observed easily from a plane ascending or descending through them. In many cases the pilot can also make an estimate of the ratio of the meteorological range above and below the boundary. Proficiency in describing the stratification of the atmosphere is acquired very quickly by any flyer, once he understands what to look for. Moreover, stratification should correlate with other meteorological conditions, and ex-

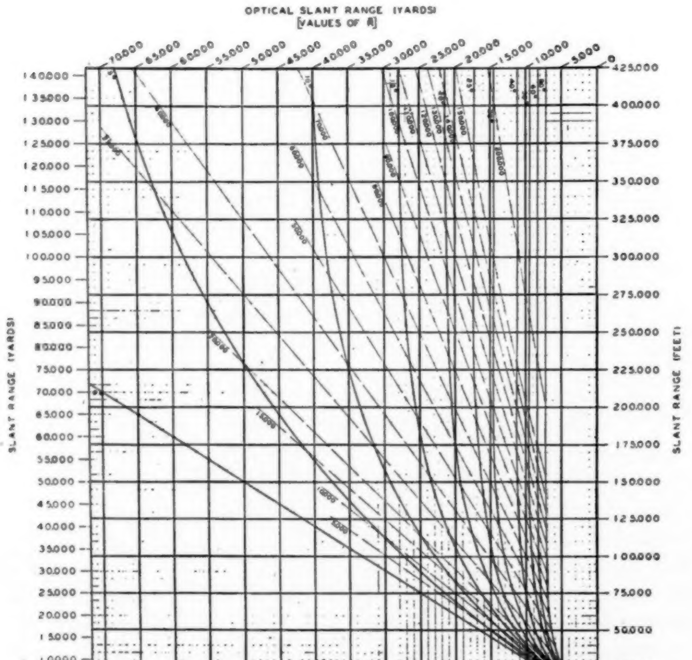

FIG. 6. Optical slant-range diagram for the optical standard atmosphere. Solid curves represent the relation between $\bar{R}$ and R for various sight-path elevation angles θ. Broken lines represent loci of equal altitude expressed in feet.

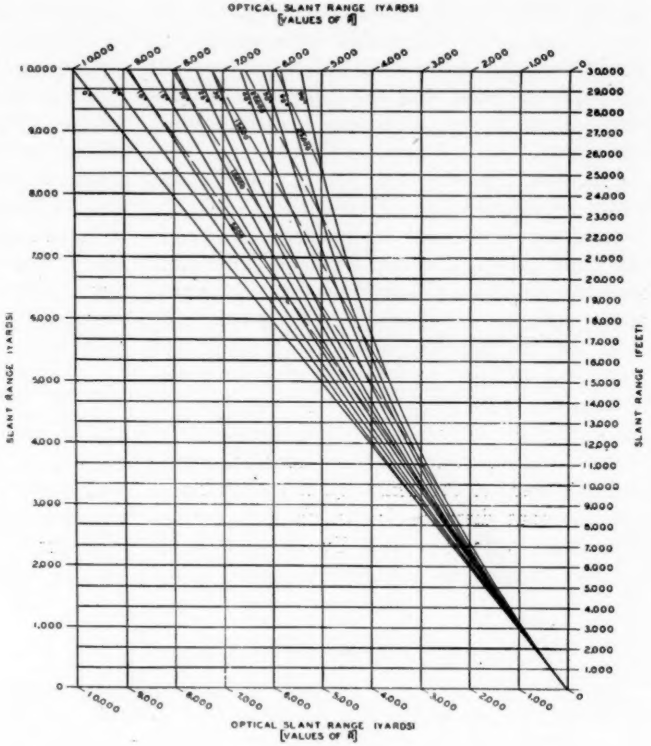

FIG. 7. Optical slant-range diagram similar to Fig. 4, but adapted to the solution to problems involving short slant ranges.

perience may enable very intelligent guesses to be made by an observer on the ground. Statistical information concerning the frequency of occurrence of common stratification conditions in a given locality can be accumulated in the same manner as other meteorological data.

Experience in drawing curves of modified slope on the optical slant-range diagram is quickly acquired with practice. Freehand curves are usually as precise as are warranted by estimates of the ratio of meteorological ranges within the strata. Often straight lines are sufficient approximations for curves of very great or very small slopes. An example of the latter occurs in the case of an opaque cloud deck within which the meteorological range is very short. Usually the lower boundary of the cloud layer is sharply defined; it can be represented on the optical slant-range diagram by a horizontal straight line passing through the point on the standard curve which corresponds to the altitude of the ceiling.

Along horizontal paths of sight local variations in q may be caused by cloud shadows or by variations in the reflectivity of the natural terrains. Moreover, local wind or temperature conditions may cause β_0, τ_0, and σ_0 to vary along the path of sight. If information concerning the variations is available or if they can be estimated, $\bar{R}$ can be found by altering the shape of the curve for $\theta=0$ degrees on the optical slant-range diagram.

4. SUMMARY

A general expression (15) for the reduction of apparent contrast by the atmosphere has been obtained for the most commonly encountered outdoor visual tasks, allowance having been made for absorption and for both primary and secondary scattering. Special relations have been derived for the case of visibility upward (18), visibility downward (20), and horizontal visibility (22). In the special case of an object seen against a background of horizon sky, the apparent contrast is exponentially attenuated (19). Experimental evidence of the validity of Eq. (19) supports the belief that the contrast equations are valid for inherent contrasts at least as great as unity.

From the principle of optical equilibrium, which states that certain luminance levels are transmitted unchanged by the atmosphere, it has been shown how the terminal luminances seen by observers looking upward, downward, or horizontally are related to the optical constants of the atmosphere. In many cases they are equal to the apparent luminance of the horizon sky in specified directions (Fig. 2).

Specification of the attenuating properties of the atmosphere in terms of its contrast transmittance leads to the definition of *meteorological range* as that distance for which the contrast transmittance is two percent. Two allegedly complicating effects (the "ground-glass plate" effect and the "edge" effect) were searched for but not found.

Because all distances have been expressed in terms of the *optical slant range* $\bar{R}$, the equivalent horizontal path through a homogeneous atmosphere, the contrast reduction equations can be used along any path of sight if $\bar{R}$ can be evaluated. Whenever atmospheric conditions simulate a "standard atmosphere," this can be done by means of an optical slant-range diagram (Figs. 4 and 5). When non-standard atmospheric conditions prevail the diagram often can be modified to conform with the state of the atmosphere.

5. ACKNOWLEDGMENTS

The sustained and stimulating interest of all the personnel of NDRC Section 16.3, its Army and Navy liason officers, and many of its contractors is gratefully acknowledged. The author is indebted to the United States Weather Bureau and to the National Bureau of Standards for

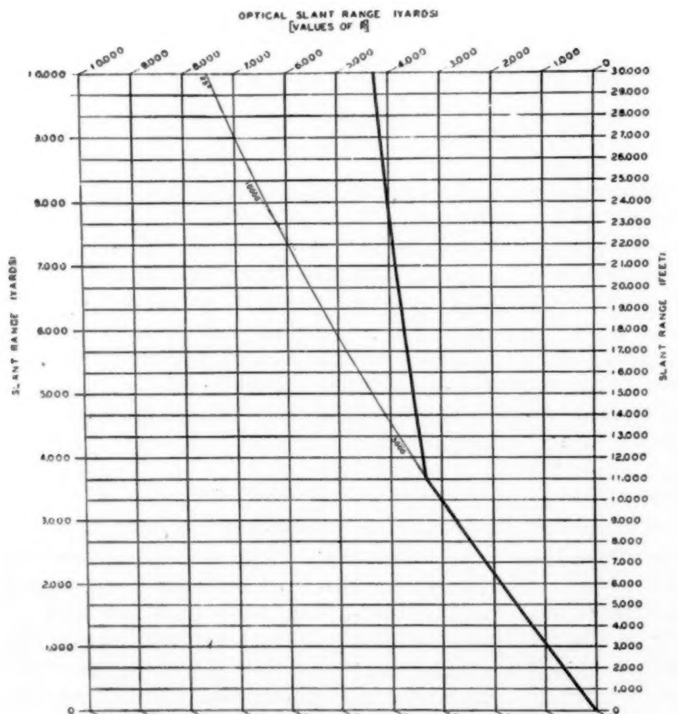

FIG. 8. Optical slant-range diagram for $\theta = 25$ degrees. Accentuated curve shows relation between $\bar{R}$ and R when the ground haze has a sharp upper boundary at 5000 feet, above which the meteorological range is five times greater than it is below the boundary.

their interest and assistance in the experimental aspects of the work. Prominent individual contributions to the experiments were made by Mr. C. A. Elford of the U. S. Weather Bureau, Mr. C. A. Douglas of the National Bureau of Standards, Dr. D. L. MacAdam of the Eastman Kodak Company, and Messrs. W. P. Greenwood and H. H. Lane of the Louis Comfort Tiffany Foundation. Flight facilities were provided by the Army Air Forces.

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A Method of Spectrographic Analysis of Impurities in Materials for Oxide Coating of Thermionic Cathodes*

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A method for the quantitative spectrographic analysis of the materials used in oxide coated thermionic cathodes for Fe, Cu, Pb, Mg, Mn, and Al is described. The materials consist of barium-strontium carbonate or barium-strontium-calcium carbonate. The basic raw materials consist of barium nitrate, strontium nitrate, and calcium nitrate. The method employs briquetted samples and a high voltage alternating current arc. Fifteen samples can each be analyzed for six elements in two hours with an elapsed time of eighty seconds per determination.

INTRODUCTION

In the manufacture of radio tubes and fluorescent lamps, the purity of the cathode coating plays a very important role in the proper functioning of the electronic device. The coating consists of a mixture of barium and strontium carbonates, or barium, strontium and calcium carbonates, which are suspended in a suitable vehicle, coated on the cathode, and reduced to the oxide during exhaust of the tube or lamp. The basic raw materials used in the preparation of the carbonates are barium, strontium, and calcium nitrates. The final carbonates can be prepared in two ways; they can each be precipitated from the nitrate with ammonium carbonate, dried, and blended together in the proper proportions, or they can be mixed in the proper proportion in a nitrate solution and coprecipitated as the carbonate.

The presence of minute impurities in the coating can have a "poisoning" effect on electron emission and, therefore, proper control in manufacturing is imperative. The present paper deals with a method of control analysis for iron, copper, lead, magnesium, manganese and aluminum in the above materials. The working ranges of the procedures are shown in Table I.

Our aim was to develop analytical procedures, for those impurities listed, which would have the following characteristics: (1) simplicity of use; (2) a minimum of time required to obtain results; (3) accuracy of the order of ± 10 percent of the amount determined; (4) concentrational sensitivity sufficient to follow purity studies.

Simplicity of use means, here, the direct handling of samples on receipt in the laboratory. Chemical manipulation is avoided if possible, but when necessary it should be kept as simple as possible. A minimum of time required to obtain results can be approached by determining as many as possible (preferably all) of the elements from a single exposure. The use of a common internal standard line contributes materially to this end. Accuracy of the order of ± 10 percent of the amount determined may be hampered somewhat if adherence to the other characteristics is maintained. Concentrational sensitivity sufficient to follow purity studies requires that the method be readily capable of

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TABLE I.

Material	Element line	Reference line	Concentration range—percent
Ba-Sr-Ca CO_3 (As nitrate)	3247.5 Cu	3253.0 Ba	.00001 –.01
	3092.7 Al	3253.0 Ba	.001 –.05
	3020.6 Fe	3253.0 Ba	.0001 –.1
	2833.0 Pb	3253.0 Ba	.0005 –.06
	2794.8 Mn	3253.0 Ba	.00001 –.003
	2802.7 Mg	2641.3 Ba	.0001 –.02
Ba-Sr-CO_3 (As nitrate)	3247.5 Cu	3253.0 Ba	.00005 –.002
	3092.7 Al	3253.0 Ba	.0001 –.1
	3020.6 Fe	3253.0 Ba	.001 –1.0
	2833.0 Pb	3253.0 Ba	.001 –1.0
	2794.8 Mn	3253.0 Ba	.00001 –.001
	2802.7 Mg	2641.3 Ba	.0003 –.03
$Ba(NO_3)_2$	3247.5 Cu	3253.0 Ba	.00001 –.01
	3092.7 Al	3253.0 Ba	.0001 –.03
	3020.6 Fe	3253.0 Ba	.0001 –.1
	2833.0 Pb	3253.0 Ba	.0004 –.04
	2794.8 Mn	3253.0 Ba	.00001 –.002
	2802.7 Mg	2641.3 Ba	.0004 –.3
$Sr(NO_3)_2$	3247.5 Cu	3371.0 Sr	.00001 –.005
	3092.7 Al	3371.0 Sr	.0002 –.06
	3020.6 Fe	3371.0 Sr	.0001 –.06
	2833.0 Pb	3371.0 Sr	.0002 –.06
	2794.8 Mn	3371.0 Sr	.00005 –.005
	2802.7 Mg	3371.0 Sr	.00001 –.01
$Ca(NO_3)_2 \cdot 4H_2O$	3247.5 Cu	3274.6 Ca	.00001 –.003
	3092.7 Al	3274.6 Ca	.00015 –.02
	3020.6 Fe	3274.6 Ca	.0001 –.05
	2833.0 Pb	3274.6 Ca	.0001 –.06
	2794.8 Mn	3274.6 Ca	.000019–.0045
	2802.7 Mg	3274.6 Ca	.0001 –.02

Wave-lengths taken from Harrison's *Massachusetts Institute of Technology Wavelength Tables* (John Wiley and Sons, Incorporated, New York, 1939).

exceeding the normal lower limit of each analytical curve. We shall now discuss the method of preparing standards, the equipment used, as well as the method of attacking such a problem in this laboratory. A summary of the adopted analytical procedure is presented also.

PREPARATION OF STANDARDS AND SAMPLES

The preparation of standard samples consists in preparing a nitrate solution of a known weight of base material and adding measured volumes of standard impurity solutions to suitable aliquots of this solution. Ordinarily, 30 grams of base material per 1000 milliliters of nitric acid solution are used. A stock solution for each impurity containing 1 gram of impurity per 100 milliliters of nitric acid solution is prepared. The quantity of nitric acid used in each instance is simply that required to effect solution, excess being avoided. The impurity stock solutions are diluted for use where necessary. Fifty milliliter portions of the base material solution are taken and suitable volumes of the impurity stock solutions added to obtain the desired concentrations. The final solutions are then dried in an oven, scraped from the beakers with a spatula and ground with an agate mortar and pestle. The standards are then ready for use. Seven standards were prepared for each base material with the additions covering the ranges listed in Table I.

The samples are handled directly if already in the nitrate form. If not, they are converted to the nitrate (dried in an oven, etc.) and then analyzed.

DESCRIPTION OF EQUIPMENT

A Bausch and Lomb large Littrow spectrograph is used with a setting to include wavelength. The arc source consists of a 37.5 kva transformer delivering 15 amperes at 2200 volts a.c. The current is controlled in the secondary by means of a bank of "Glo-Coil" heaters (1000 watts each). Suitable switches permit the selection of any current value from 2 to 15 amperes in steps of approximately 0.3 ampere. The microphotometer, arc stand, developing machine, and arc source were designed and built in the laboratory to best serve our needs for spectrographic analysis. Pertinent points concerning these units are that the arc stand has water-cooled clamps and accurate control of the arc gap, while the developing machine has a two-directional rocking motion, out of phase, so that the solutions traverse ten different paths before repeating. The microphotometer operates on principles similar to the unit described by Vincent and Sawyer.[1] The microphotometer lamp voltage is maintained constant by a voltage regulator as described previously.[2] The construction of the equipment is such that it is sturdy, reliable and convenient, which has reduced the maintenance required to a minimum.

PHOTOGRAPHY AND MICROPHOTOMETRY

Eastman type 33 plates are used throughout. They are developed in D-8 developer (2:1) for 60 seconds, hardened in a chrome alum hardening bath for 75 seconds, fixed in F-8 fixer for 75 seconds, washed under a stream of water for 3 minutes and dried rapidly.[3] A plate can be

[1] H. B. Vincent and R. A. Sawyer, "A new microphotometer," J. Opt. Soc. Am. **31**, 639 (1941).

[2] S. L. Parsons, "Voltage regulator for a densitometer lamp," J. Opt. Soc. Am. **32**, 153 (1942).

[3] H. B. Vincent and R. A. Sawyer, "Rapid processing of photographic plates for routine spectrographic analysis," Spectrochimica Acta 1, 131 (1939).

completely processed in 12 minutes time by this method. We have filed plates for many years, processed in this manner, with no deterioration indicative of inadequate processing. The emulsion was calibrated using the method of Sawyer and Vincent.[4,5] It consists of photographing the spectrum of a standard iron arc operated under reproducible conditions on each plate and reading the blackening of ten selected iron lines. The blackenings of these lines are then plotted against the log intensity values previously assigned each line from measurements made with a rotating step sector. This curve is then used to obtain the log intensity ratio of each impurity spectral line to the internal standard spectral line. Using the standard samples prepared as previously described, an analytical curve is then set up relating log intensity ratio and log percent concentration. For unknown samples, the log I ratio is obtained and the percent concentration is read from the analytical curve. A sample of known concentration is photographed with each group of unknowns to correct for any lateral shifts in the analytical curves. We have found that the reduction of data is appreciably eased with the aid of a calculating board.

PROCEDURE

The spectrograph is set for the spectral region 2650A to 3750A, and the arc stand is positioned 62.5 centimeters from the slit so that the arc image is focussed on the collimator lens of the spectrograph by means of a lens placed immediately in front of the slit. This method of focusing permits rigid positioning of the arc stand. The only adjustment necessary is the electrode separation which can be obtained by moving only the upper electrode. The spectrograph slit is set at a width of 20 microns and a height of 5 millimeters. These settings result in an area of spectral line adequate to keep the microphotometer error to a minimum.

The most suitable manner of introducing the samples into the arc was studied rather thoroughly. Various shapes of graphite electrodes, in which the sample was placed in a crater, were tried. The high current values necessary for adequate concentrational sensitivity resulted in a troublesome background level. An investigation of briquetting techniques seemed logical and proved very worth while. As appreciably more sample can be placed in the arc, it is possible to use lower current values and keep the background to a tolerable level. The use of briquetted samples resulted in greatly improved reproducibility and high concentrational sensitivity.

The briquettes used are 0.1865 inch diameter, 0.125 inch high and consist of approximately 200 milligrams of sample and 4 milligrams of graphite powder. The graphite powder serves to lubricate the mold and to prevent sticking, which can be very troublesome with these powders. The supporting electrodes are graphite rods 0.750 inch long by 0.250 inch diameter. The crater is 0.1875 inch diameter by 0.125 inch deep with straight sides and a flat bottom. The outer wall is reduced so that the final wall thickness of the crater is 0.005 inch. The counter electrode has a

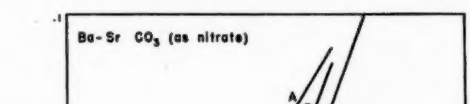

FIG. 1. Typical analytical curves.

[4] R. A. Sawyer and H. B. Vincent, "Developments in the rapid spectrographic analysis of iron and steel," Proc. 5th Summer Conf. on Spect. and its Applications (1938).

[5] S. L. Parsons, "Spectrographic determination of thorium in tungsten filament wire," J. Opt. Soc. Am. **33**, 659 (1943).

bevelled tip. The electrodes are shaped by means of a cutter, made for the purpose, mounted on a motor shaft. The graphite powder is obtained by crushing the graphite rod with a mortar and pestle; very fine powder is unsatisfactory for this purpose.

In order to select the optimum conditions for each variable, the following were studied carefully:[5, 6]

1. The effect of burning time on log I ratio.
2. The effect of varying the arc current on log I ratio.
3. The effect of varying the arc gap on log I ratio.

These effects were studied on each of a number of line pairs and in addition analytical curves were plotted and accuracy checks run for each line pair. The line pairs selected, as a result of these tests, are shown in Table I.

TABLE II.

Material	Impurity	No. of determinations	Deviation
Ba-Sr-Ca CO_3 (As nitrate)	Cu	30	8 percent
	Al	15	13 percent
	Fe	30	17 percent
	Pb	30	7 percent
	Mn	30	6 percent
	Mg	30	6 percent
Ba-Sr CO_3 (As nitrate)	Cu	30	3 percent
	Al	15	13 percent
	Fe	30	13 percent
	Pb	30	4 percent
	Mn	30	8 percent
	Mg	30	4 percent
$Ba(NO_3)_2$	Cu	15	7 percent
	Al	15	5 percent
	Fe	15	13 percent
	Pb	15	13 percent
	Mn	15	10 percent
	Mg	15	5 percent
$Sr(NO_3)_2$	Cu	30	8 percent
	Al	30	7 percent
	Fe	30	14 percent
	Pb	30	14 percent
	Mn	30	13 percent
	Mg	30	20 percent
$Ca(NO_3)_2 \cdot 4H_2O$	Cu	30	9 percent
	Al	30	8 percent
	Fe	30	17 percent
	Pb	30	12 percent
	Mn	30	11 percent
	Mg	30	8 percent

[6] E. A. Boettner and C. F. Tufts, "A study concerning characteristics of the high voltage a.c. arc," J. Opt. Soc. Am. **37**, 192 (1947).

The foregoing tests led to the following conclusions:

1. Arcing conditions of 7 amperes, 2200 volts a.c., are the most desirable, in this instance, from the standpoint of accuracy, sensitivity, and extent of background produced.
2. An exposure time of 30 seconds is adequate, no pre-burn being indicated. In fact, preburn is undesirable in this instance.
3. A 2-millimeter arc gap is satisfactory; variations from 1.5 to 2.5 millimeters showed no change in the log I ratio obtained which provides adequate latitude in adjusting the gap.
4. The number of lines to be read can be kept to a minimum by using a common internal standard line, in most instances. Using a common internal standard line, the accuracy has proved to be adequate for the intended purpose. While an improvement in accuracy could be obtained, in a few instances, by using different internal standard line, it was decided to sacrifice this in favor of realizing a saving in control lab time.

After the final selection of line pairs, arcing conditions, sampling method, etc., the final analytical curves were prepared and thorough tests made on the procedures to determine accuracy and reproducibility. A typical analytical curve (barium-strontium carbonate as the nitrate) is shown in Fig. 1.

The percent standard deviation was determined from the formula,

$$(\Sigma d^2/n-1)^{\frac{1}{2}}/\text{ave. conc.}\times 100,$$

where d = deviation from average concentration and n = number of determinations. The results of applying the formula for a series of repeat analyses are shown in Table II. The concentrations of the elements in the samples used for these tests fell within the ranges covered by the analytical curves.

In our company, we have three widely separated chemical laboratories through which analytical results can be checked. This companywide correlation is mutually beneficial in that it insures similar results, no matter which laboratory obtains them. Cooperative effort among the three chemical laboratories and two spectrochemical laboratories is maintaining excellent agreement on analytical results, as well as keeping everyone on their toes. All of the procedures discussed here have been correlated in this manner. This has enabled us to be more certain of our standards and we feel that the analyses obtained are accurate determinations of the amount of impurity present. Another spectrographic laboratory, with similar equipment, is located in the Tungsten and Chemical Division of Towanda, Pennsylvania. These procedures have been installed at that laboratory by simply using the prescribed operating instructions resulting from this investigation. Agreement with the research laboratory analyses was obtained immediately. This practice has been followed with many of the spectrographic procedures developed in the research laboratory with surprising ease of transfer, which appears to be a good test of the quality of a particular procedure.

ANALYSIS PROCEDURE

The samples are converted to the nitrate if not already in that form. Briquettes are prepared and placed in graphite supporting electrodes and burned at 7 amperes, 2200 volts a.c., for 30 seconds. A slit width of 20 microns, a slit height of 5 millimeters, and a 2-millimeter arc gap are used throughout. An iron spectrum is photographed on each plate to provide the emulsion calibration curve. A sample of known concentration is also photographed on each plate to correct for any lateral shifts in the analytical curves. The plates are processed rapidly and the blackenings of the selected lines read with the microphotometer. The microphotometer readings are converted to concentration values with the aid of a calculating board. Fifteen samples can be analyzed for Fe, Cu, Pb, Mg, Mn, and Al in two hours or an elapsed time of eighty seconds per determination, which has met our requirement of fairly rapid results in this instance. A faster and more convenient briquetting device than that used here would materially decrease the time required.

ACKNOWLEDGMENT

The authors are particularly indebted to Dr. R. M. Bowie, Manager of Research, for constant advice and encouragement, to Miss Jean E. Kinnear, spectrochemist at the Towanda laboratory, for her assistance in testing the procedures, and to J. B. Merrill, General Manager of the Tungsten & Chemical Division, for his interest and enthusiasm which led to early application of the procedures to manufacturing problems. We are grateful to the various laboratories of Sylvania Electric Products, Incorporated for their assistance throughout the investigation.

The Sensitivity Performance of the Human Eye on an Absolute Scale*

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(Received August 11, 1947)

An absolute scale of performance is set up in terms of the performance of an ideal picture pickup device, that is, one limited only by random fluctuations in the primary photo process. Only one parameter, the quantum efficiency of the primary photo process, locates position on this scale. The characteristic equation for the performance of an ideal device has the form

$$BC^2\alpha^2 = \text{constant}$$

where B is the luminance of the scene, and C and α are respectively the threshold contrast and angular size of a test object in the scene. This ideal type of performance is shown to be satisfied by a simple experimental television pickup arrangement. By means of the arrangement, two parameters, storage time of the eye and threshold signal-to-noise ratio are determined to be 0.2 seconds and five respectively. Published data on the performance of the eye are compared with ideal performance. In the ranges of $B(10^{-6}$ to 10^2 footlamberts), $C(2$ to 100 percent) and $\alpha(2'$ to $100')$, the performance of the eye may be matched by an ideal device having a quantum efficiency of 5 percent at low lights and 0.5 percent at high lights. This is of considerable technical importance in simplifying the analysis of problems involving comparisons of the performance of the eye and man-made devices. To the extent that independent measurements of the quantum efficiency of the eye confirm the values (0.5 percent to 5.0 percent), the performance of the eye is limited by fluctuations in the primary photo process. To the same extent, other mechanisms for describing the eye that do not take these fluctuations into account are ruled out. It is argued that the phenomenon of dark adaptation can be ascribed only in small part to the primary photo-process and must be mainly controlled by a variable gain mechanism located between the primary photo-process and the nerve fibers carrying pulses to the brain.

INTRODUCTION

THE designer of picture pickup devices such as television pickup tubes, photographic film and electron image tubes is faced steadily with the problem of comparing the performance of these devices with the performance of the human eye. This is especially true for comparisons of sensitivity. Neither television pickup tubes nor photographic film match the ability of the eye to record pictures at very low scene luminances. Film ceases to record at a scene luminance of a few footlamberts, and present television pickup tubes at a few tenths of a footlambert. (Lens diameters and exposure times are assumed equal to those of the eye.) The eye, however, still transmits a picture at 10^{-6} footlambert. This is a striking discrepancy, especially when it is known that eye, film and pickup tube each require about the same number of incident quanta to generate a visual act. By visual act is meant a threshold visual sensation for the eye, the rendering of a photographic grain developable in film or the release of a photo-electron in a television pickup tube. This number of incident quanta is in the neighborhood of 100. The sources of this discrepancy will be discussed later. For the present, the discrepancy is introduced and emphasized for the following reason. Since television pickup tubes and photographic film are already limited in their performance by more or less fundamental statistical fluctuations (noise currents in pickup tubes and graininess in film) and since the low light performance of the eye so far outstrips that of pickup tubes and film, it is not unreasonable to inquire whether the performance of the eye also is limited by statistical fluctuations.

The purpose of this paper is, in fact, to lay out clearly the absolute limitations to the visual process that are imposed by fluctuation theory and to compare the actual performance of the eye with these limitations. The gap, if there is one, between the performance to be expected from fluctuation theory and the actual performance of the eye is a measure of the "logical space" within which one may introduce special mechanisms, other than fluctuations, to determine its performance. These special mechanisms can only contract the limits already set by fluctuation theory. This point is especially important because it restricts the freedom with which one can introduce such assumptions as: (1) rods or cones

* Presented in part at the November 1945 meeting of the American Physical Society in New York.

with variable thresholds of excitation, (2) an absorption coefficient for the retina that varies with scene luminance or (3) photo-chemical reaction rate equations with arbitrary coefficients.

The following discussion begins with a description of ideal performance, that is, performance limited only by statistical fluctuations in the absorption of light quanta. Next an experimental realization of ideal performance is introduced in the form of a special television pickup arrangement. The performance data for the eye is then compared with ideal performance and finally some implications of this comparison are discussed.

It must be emphasized that this discussion is concerned primarily with the low light end of the light range over which the eye operates. It is here that fluctuation limitations would be expected to be the dominant factor. At very high lights other limitations set in, as for example, the finite structure of the retinal mosaic, or the limited traffic carrying capacity of the optic nerve fibers. Important as these factors are for a complete understanding of the eye, they do not constitute, as do statistical fluctuations, an absolute limit to the possible performance of the visual process. They are the particular boundary conditions pertaining to the eye, which, in another device or in an "improved eye," might take on other values. The light range considered here is still the larger part of the total light range of the eye, namely, from 10^{-6} to 10^2 footlamberts. The excluded range is 10^2 to 10^4 footlamberts.

Also the discussion is confined, except for a few remarks on color, to the sensitivity performance of the eye for white (as opposed to colored) test patterns.

PERFORMANCE OF AN IDEAL PICTURE PICKUP DEVICE

An ideal picture pickup device is defined to be one whose performance is limited by random fluctuations in the absorption of light quanta in the primary photo-process. Each absorbed quantum is assumed to be observable in the sense that it may be counted in the final picture. From well known statistical relations, an average absorption of N quanta will have associated with it deviations from the average whose root mean square value is $N^{1/2}$. These deviations are a measure of the accuracy with which the average number N may be determined. They also control the smallest change in N that may be detected. Thus if this smallest change is denoted by ΔN:

$$\Delta N \sim N^{1/2} \tag{1}$$

or

$$\Delta N = kN^{1/2} \tag{1a}$$

where k is a constant to be determined experimentally. k is called the threshold signal-to-noise ratio.

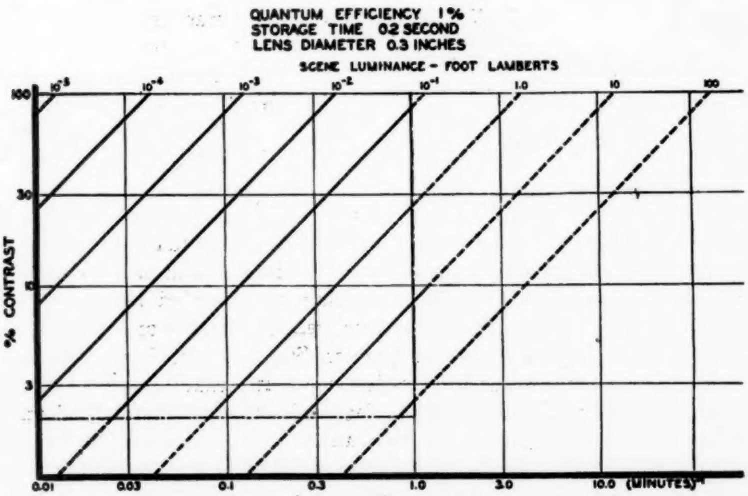

FIG. 1. Performance of ideal pickup device. The experimentally determined value, 5, of threshold signal-to-noise ratio was used to compute these curves.

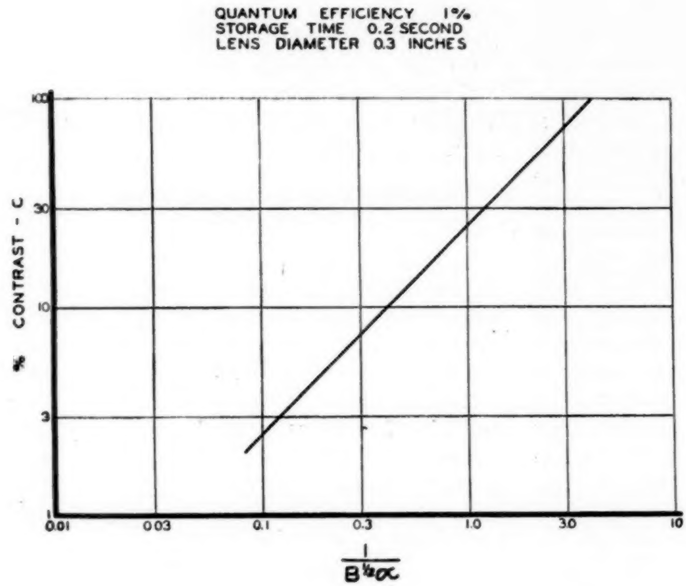

FIG. 2. Performance curve for ideal pickup device. A reduced plot of the curves in Fig. 1.

Let the average number of quanta, N, be absorbed in an element of area of side length, h, and in the exposure time of the pick-up device. Then N/h^2 is proportional to the luminance of the original scene and ΔN to the threshold change in luminance and we may write

$$\text{scene luminance} \equiv B \sim (N/h^2), \tag{2}$$

$$\text{threshold contrast} \equiv C = \Delta B/B \times 100\% = \Delta N/N \times 100\% \sim 1/N^{\frac{1}{2}}. \tag{3}$$

Combining Eqs. (2) and (3) we get:

$$B \sim 1/C^2h^2, \tag{4}$$

$$B \sim 1/C^2\alpha^2, \tag{4a}$$

or

$$B = \text{constant } (1/C^2\alpha^2), \tag{4b}$$

where α is the angle subtended by h at the lens. Equation (4b) is the characteristic equation for the performance of an ideal picture pickup device. It is based on the simplest and most general assumptions regarding the visual process. Since no special mechanism has been called upon, it applies equally well to chemical, electrical or biological processes of vision. The constant factor includes among other constants, the storage time, quantum efficiency and optical parameters of the particular device. When two ideal devices are compared for performance under equivalent conditions, the only distinguishing parameter is their respective quantum efficiencies.

Equation 4b provides the threshold value of any one of the variables when the other two are arbitrarily specified. Thus Fig. 1 shows a plot on a log−log scale of threshold contrast as a function of visual angle for various fixed values of the scene luminance. In Fig. 2, threshold contrast is plotted as a function of $1/(B^{\frac{1}{2}}\alpha)$ and, as expected from Eq. (4b), all of the performance data of Fig. 1 collapse into a single straight line. The location of this line determines the constant in Eq. (4b) and from this constant the quantum efficiency of the device may be computed (see Eq. 5).

It should be clear that there is nothing in the fluctuation theory used to derive Eq. (4b) that would prevent the lines in Fig. 1 from being extended indefinitely to the right toward small angles or indefinitely downward toward low contrasts. It should also be clear, on the other hand, that any actual physical device will impose such limitations. The smallest angle that can be resolved may be limited either by structure in the surface on which the optical image is focused or eventually by diffraction effects in the optical focus itself. Also any actual physical device cannot generate arbitrarily high signals as would be required if the lines in Fig. 1 were extended to arbitrarily small contrasts. Both these limitations have no necessary connection with fluctuation theory and serve merely to define the boundaries within which such a theory may be applied. Such boundaries may be shown, for example, as in Fig. 1 by the two dash-dot lines. The lines represent asymptotic values approached by an actual device under high light conditions. For this reason, data plotted for an actual device would be expected to bend away from their theoretically straight lines as they approach the dash-dot boundaries.

The complete characteristic equation with the constant factor written out is

$$B=5(k^2/D^2t\theta)(1/\alpha^2C^2)\times10^{-3}\text{ footlambert,}\qquad(5)$$

where the symbols have these meanings and units:

k—threshold signal-to-noise ratio [see Eq. (1a)]

D—diameter of the lens (inches)

t—exposure time (seconds)

θ—quantum yield ($\theta=1$ means 100 percent quantum efficiency)

α—angular size of the test object in minutes of arc

C—percent contrast of the test object [i.e., $C=(\Delta B/B)\times100$ percent]

The constant factor is derived as follows. In place of Eq. (2) we write (see Fig. 3):

$$N=\theta N_0tl^2\sin^2\phi,\qquad(6)$$

where N_0 is the total number of quanta emitted per ft.² of the scene per second according to a Lambert distribution. Now since

$$l\doteqdot(d/F)h,\quad\text{and}\quad\sin\phi\doteqdot D/2d,$$

we can write

$$N=\tfrac{1}{4}\theta N_0tD^2(h^2/F^2)=1.4\theta N_0tD^2\alpha^2\times10^{-10},\qquad(7)$$

where α is expressed in minutes of arc and D in inches. Using the equivalence, one lumen of white light $=1.3\times10^{16}$ quanta per second,

$$N_0/1.3\times10^{16}=B\text{ footlamberts}$$
$$N=2\theta BtD^2\alpha^2\times10^6,$$

and

$$B=5(N/D^2t\theta\alpha^2)\times10^{-7}\text{ footlambert.}\qquad(8)$$

From Eqs. (3) and (1a) we get:

$$C=100k/N^{\frac{1}{2}}.\qquad(9)$$

Combining Eq. (8) and (9) we get:

$$B=5(k^2/D^2t\theta)(1/\alpha^2C^2)\times10^{-3}\text{ footlambert}\qquad(10)$$

The factor k in Eq. (5) is of special interest because its value has frequently been assumed to be unity. That is, the statement is made that a threshold signal is one that is just equal to the r.m.s. noise.** Some estimates made recently by the writer[3] and based on observations on photographic film and on television pictures lay in the

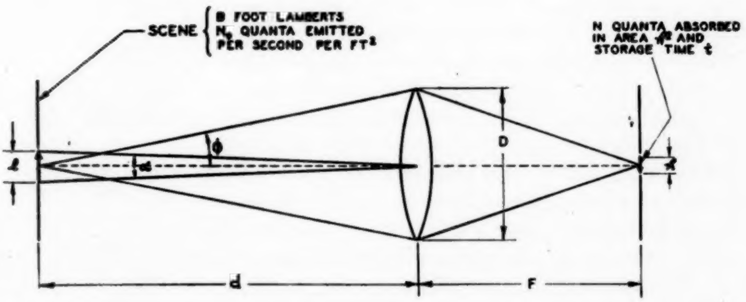

FIG. 3.

** Such an assumption, for example, was made by the writer (reference 1) and also by H. De Vries (reference 2).

[1] A. Rose, "The relative sensitivities of television pickup tubes, photographic film and the human eye," Proc. I.R.E. **30**, 295 (1942).

[2] H. DeVries, "The quantum character of light and its bearing upon threshold of vision, the differential sensitivity and visual acuity of the eye," Physica **10**, 553 (1943).

[3] A. Rose, "A unified approach to the performance of photographic film, television pickup tubes and the human eye," J.S.M.P.E. **47**, 273 (1946).

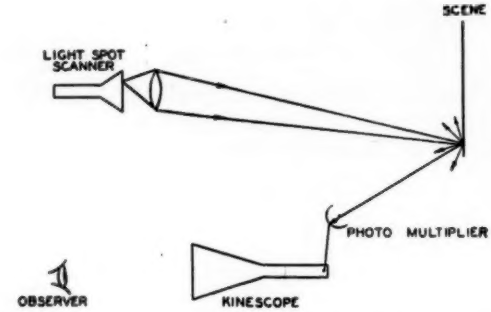

FIG. 4. Television pickup arrangement using a light spot scanner.

range of 3 to 7. Additional and more direct evidence is given in the next section that the value of k is not unity but is in the neighborhood of 5.

AN EXPERIMENTAL APPROACH TO AN IDEAL PICTURE PICKUP DEVICE

One of the oldest means of generating television pictures is the so-called light spot scanning arrangement in which the subject to be transmitted is scanned by a small sharply focused spot of light. The variable amount of light reflected from the subject is picked up by a photocell and these variations translated into beam current variations in a kinescope whose beam scans a fluorescent screen in synchronism with the first light spot. The arrangement is shown in Fig. 4. Recent developments in luminescent materials and photo multipliers have brought renewed interest in the arrangement for certain types of pick-ups.[4] On the one hand, it is especially simple and free from the spurious signals usually found in pickup tubes. On the other hand, it is limited in application to those scenes that may be conveniently illuminated by a scanning light spot. Its particular virtue for the present discussion is that it offers a close approximation to the performance to be expected from an ideal picture pickup device. The photo-cathode of the electron multiplier represents at once both the lens opening and primary photo surface of the usual pickup device. The gain in the multiplier section is sufficient to make each photo electron, liberated from the photo-cathode, visible on the kinescope screen as a discrete speck of light. That is, each quantum usefully absorbed at the primary photo-surface can be counted in the final picture. The exposure time of the system is the exposure time of the final observer (human or instrumental) that looks at the reproduced picture on the kinescope.

The special test pattern used as subject or scene for the light spot scanner is shown in Fig. 5. This test pattern is in fact a materialization of the theoretical curves in Fig. 1. The disks along any row decrease in diameter by a factor of two for each step. The disks in any column have the same diameter but vary in contrast stepwise by a factor of two. If this pattern is reproduced by a pickup device performing in accordance with Fig. 1, all of the disks to the upper left of some 45° diagonal should be visible and all of those to the lower right should not. As the illumination is increased, the diagonal demarcation between visibility and invisibility should move to the right and in particular should move from one diagonal of disks to the next for a factor of four increase in illumination.

The series of pictures shown in Fig. 6 is a series of timed exposures of the picture reproduced on the kinescope as the light spot from another cathode ray tube scanned the test pattern. For experimental convenience, the exposure time, rather than the scene luminance, was increased, since, according to Eq. (5), it is only the product Bt that is significant. The first pictures in the series show what is transmitted at exceed-

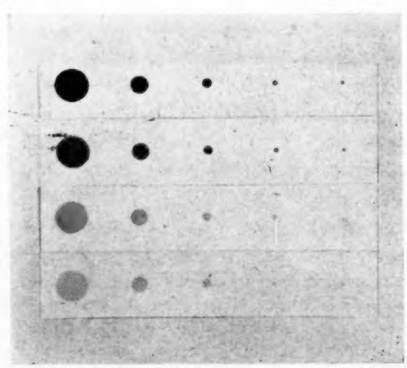

FIG. 5. Photograph of test pattern used as subject for the light spot scanner.

[4] G. C. Sziklai, R. C. Ballard and A. C. Schroeder, "An experimental simultaneous color television system, Part II: Pickup equipment," Proc. I.R.E. **35**, 862 (1947).

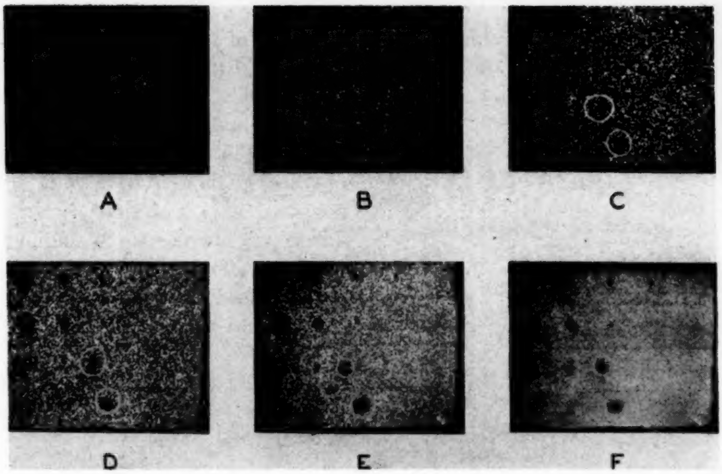

FIG. 6. Series of timed exposures of the test pattern shown in Fig. 5 as transmitted by the television pickup arrangement shown in Fig. 4. The exposure times, starting with Fig. 6a, are $\frac{1}{16}$, $\frac{1}{4}$, 1, 4, 16 and 64 seconds respectively. These exposures were chosen so that the diagonal demarcation between visibility and invisibility fell to the right of rather than on a particular diagonal of discs. Thus the smallest visible black dots are somewhat above threshold visibility. To get a short decay time, the ultraviolet emission from a special zinc-oxide phosphor scanner was used.† Two obvious blemishes that were not apparent under visible light, and have no connection with the test, are marked off by circles in Figs. 6c, d, e, and f.

ingly low scene luminance. In fact the number of "quanta" per unit area may easily be counted. As the scene luminance is increased, more and more of the pattern becomes visible.

Equation (5) and Figure 1 are quantitatively borne out by these pictures in two important respects. First, the demarcation between visibility and invisibility is, with good approximation, a diagonal. That is, the threshold contrast varies as the reciprocal angle of the test object. Second, the demarcation shifts by one diagonal for a factor of four change in scene luminance. That is, the threshold scene luminance varies as the square of the reciprocal contrast or as the square of the reciprocal angle. While the precision of the separate pictures is not high, the precision of the series is, since there are no significant cumulative or progressive departures in the large range of scene luminance covered.

The series of pictures in Fig. 6 also establishes the values of two of the parameters in Eq. (5), namely the threshold signal-to-noise ratio (k) and the exposure or storage time (t) of the eye. The threshold signal-to-noise ratio has this meaning. Take the smallest black (not grey) disk that may be seen in any one of the pictures. Transpose the outline of the disk to the neighboring white background. Count the average number of "quanta" (specks of light) within this outline. The average number of "quanta" is the signal associated with the black disk; the square root of the average number is the root mean square fluctuation,*** and the ratio of signal to r.m.s. fluctuation, also the square root of the average number, is the threshold signal-to-noise ratio. A similar operation can be carried out for any of the grey dots to obtain the same value of k. The results of this operation are that k lies in the neighborhood of 5. A more precise value of k depends on a more precise operation for determining the threshold visibility of any one of the black disks. A more pre-

† This was suggested to the writer by O. H. Schade of the RCA Victor Division, Harrison, N. J. See also R. E. Shrader and H. W. Leverenz "Cathodoluminescence Emission Spectra of Zinc-Oxide Phosphors," to be published in an early issue of the Journal of the Optical Society of America.

*** This has been roughly verified by actual counts taken on Fig. 6a.

cise value for k would, however, not depart significantly from the one given here. This is based on the fact that the range from substantial certainty of not seeing to substantial certainty of seeing is covered by a factor of four in scene luminance. This corresponds to a factor of two in the range of k values that might be selected. The interesting fact is that the threshold signal-to-noise ratio is not unity as is usually assumed but more nearly five.

The storage time of the eye is usually taken to be about 0.2 seconds. The series of photographs in Fig. 6 confirmed that choice if confirmation were needed. The visual impression of the kinescope picture matched within a factor of two the photographic exposure for 0.25 second.

To summarize this section, the series of pictures in Fig. 6 form a simple, quantitative representation of the operation of an ideal picture pick-up device.

COMPARISON OF THE PERFORMANCE OF THE HUMAN EYE WITH IDEAL PERFORMANCE

Experimental data for the human eye relating scene luminance, contrast and visual angle have been scarce. The writer[3] has already made use of the data of Connor and Ganoung[5] and of Cobb and Moss[6] to cover the range from 10^{-4} to 10^{2} footlamberts. These data are reproduced in Figs. 7 and 8 and are to be compared with Figs. 1 and 2. As in the previous use of the data, values of α less than two minutes of arc and values of contrast less than two percent were omitted. These points are close to the absolute cut-offs of $\frac{1}{2}$ minute of arc and $\frac{1}{2}$ percent contrast set by other than fluctuation limitations and would be expected to depart from the theoretical curves of Fig. 1.

Recently a more complete and thorough investigation of visual performance has appeared by Blackwell.[7] The points in Figs. 9 and 10 were computed from Blackwell's data for grey disks on a white background. In order to plot both Figs. 8 and 10, Reeves'[8] data on pupil diameter versus scene luminance were used.

In Fig. 7, the data have been approximated by lines of 45° slope in accordance with Eq. (4b). The fit is close enough to be significant. The same degree of fit is not, however, present in Black-

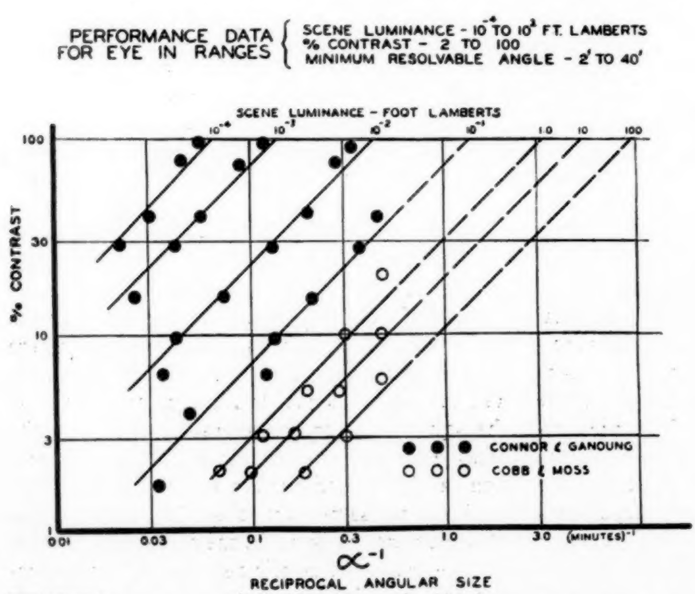

FIG. 7. The solid lines with 45° slope are approximations to the experimental data by ideal performance curves.

[5] J. P. Connor and R. E. Ganoung, "An experimental determination of visual thresholds at low values of illumination," J. Opt. Soc. Am. **25**, 287 (1935).
[6] P. W. Cobb and F. K. Moss, "The four variables of visual threshold," J. Frank. Inst. **205**, 831 (1928).
[7] H. R. Blackwell, "Contrast thresholds of the human eye," J. Opt. Soc. Am. **36**, 624 (1946).
[8] P. Reeves, "The response of the average pupil to various intensities of light," J. Opt. Soc. Am. **4**, 35 (1920).

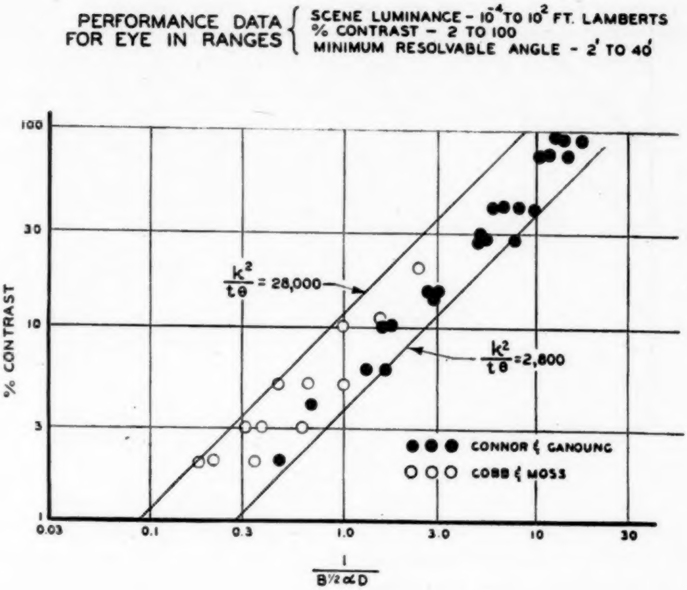

FIG. 8. A reduced plot of the data in Fig. 7. The two solid lines are computed from Eq. 5 for an ideal device

well's data in Fig. 9. Here the 45° lines are drawn tangent to the best performance at each value of scene luminance. The data in each case curve away from the straight lines. The degree of fit is still, however, sufficiently good for many engineering purposes. It is also sufficiently good to draw significant conclusions regarding the mechanism of the eye, as will be discussed below. In comparing Figs. 7 and 9 with Fig. 1 it is to be noted that the correction for the variation in pupil diameter has not yet been introduced. Such correction is introduced in Figs. 8 and 10.

In Figs. 8 and 10 the data are re-plotted as in Fig. 2. If the quantum efficiency, exposure time and threshold signal-to-noise ratio of the eye were invariant with scene luminance, and if the performance of the eye were limited by fluctuations in the absorption of light quanta, the data in Figs. 8 and 10 should all lie along a single straight line. The fact that the data do not lie along a single straight line but have some spread is a measure of the departure from one or more of the above conditions. Before discussing these departures it is well to note that Blackwell's data are substantially contained between the same two straight lines as are the data of Connor and Ganoung and Cobb and Moss.

The two straight lines that bracket the data both in Fig. 8 and in Fig. 10 are labelled $k^2/t\theta = 2800$ and $k^2/t\theta = 28{,}000$. Equation (5) was used to compute these values. The significance of these lines may be indicated as follows. If one arbitrarily assumed that k and t were invariant with scene luminance and that the performance of the eye were limited by fluctuations and took for k and t the values 5 and 0.20 respectively, then one would conclude that all of the data contained within these lines could be represented by an ideal picture pickup device having a quantum efficiency between 0.5 and 5 percent. On the one hand, this is a large spread in quantum efficiency; on the other hand, even this large spread severely limits the choice of mechanisms used to explain the phenomenon of dark adaptation, the latter covering a range of "apparent sensitivities" of over a thousand to one.

If, now, k and t instead of being assumed constant, were allowed to vary with scene luminance, their most reasonable direction of variation would be such as to reduce the range of variation of θ, the quantum efficiency. So also, if mechanisms other than fluctuations in the absorption of light quanta are used to describe the performance of the eye, these mechanisms, because they would be introduced at the high light end, would tend to reduce the range of variation of θ.

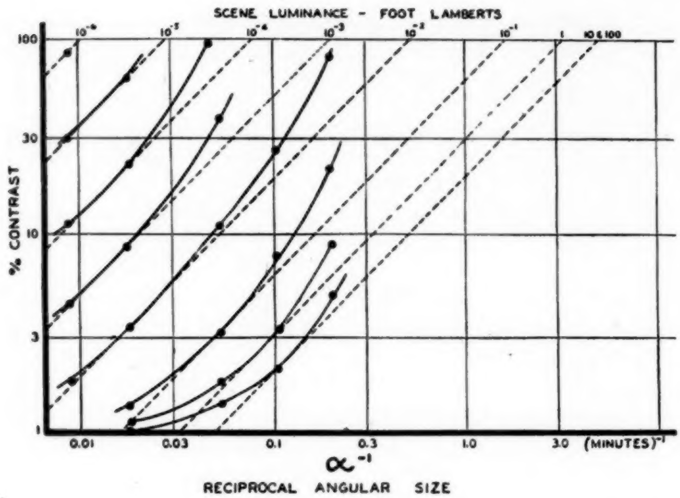

FIG. 9. Performance data for eye (computed from Blackwell). The dotted lines with 45° slope are ideal performance curves drawn tangent to the best observed performance at each value of scene luminance.

In brief, a factor of ten represents the maximum variation that the quantum efficiency of the eye undergoes in the range of 10^{-6} to 10^{2} footlamberts.

GENERAL DISCUSSION

A. Problems of Engineering Importance

The fact that the bulk of the performance data of the eye can be simply summarized by the performance of an ideal picture pickup device operating with a quantum efficiency of 5 percent at low lights and 0.5 percent at high lights is of considerable technical convenience. The ranges of the three parameters are:

scene luminance	10^{-6} to 10^{+2} footlamberts
percent contrast	2 to 100
visual angle	2′ to 100′

The types of problems that are clarified by this approach are: specification of the performance of television pickup tubes that are designed to replace the human eye; estimate of the factor by which pick-up tubes should exceed the eye in performance when the reproduced picture is viewed at a higher luminance than the original scene; estimate of the maximum gain in intelligence that may be obtained by any picture pickup device interposed between the eye and the scene; the setting up of criteria for the visibility of noise in a television picture or of graininess in photographic film; and finally, the ordering of the performance of present television pickup tubes and film relative to that of the eye. Two of these problems will be discussed briefly.

If the eye may be treated as an ideal pickup device, the criterion of threshold noise visibility is simple. It is that the signal-to-noise ratio associated with an element of area of the retina be approximately equal to the signal-to-noise ratio associated with the same element of area in the original scene in which noise is to be observed. Thus, in a series of tests in which pictures similar to those in Fig. 6 were directly viewed on a kinescope, it was found that the noise in these pictures could be reduced to threshold visibility by interposing a neutral filter between the eye and the kinescope. The transmission of the neutral filter was such that, at threshold, the number of white specks per unit area per unit time on the kinescope face was approximately equal to the number of light quanta absorbed by the retina from the same area per unit time. A quantum efficiency of 0.5 percent was used for this computation. It is probably more significant to apply the same type of analysis to data already published, as for example, in the paper by Jones and Higgins[9] on the graininess of photographic film. Table I, column 1 shows the values of signal-to-noise ratio measured by Jones and Higgins for several widely different types of film and for a test area 40 microns in diameter on the film. In

[9] L. A. Jones and G. C. Higgins, "Photographic granularity and graininess," J. Opt. Soc. Am. **36**, 203 (1946).

olumn 2 are given the computed values of signal-o-noise ratio for the same test area at the retina under what they call threshold conditions for eeing graininess. To compute column 2, a quantum efficiency of 0.5 percent was assumed for he eye as well as a pupil diameter of 4 millimeters and a storage time of 0.2 second.

The large discrepancy between the low light performance of the eye and that of present television pickup tubes and photographic film was referred to at the beginning of this paper. Its origin is this. The eye appears to act like an ideal device over a large range of scene luminances. That is, as the scene luminance is decreased the signal received by the retina falls linearly while the noise associated with the signal falls as the square root of the scene luminance. And these relations hold even down to 10^{-6} footlambert. The same relations hold for pickup tubes and film but usually only over the relatively narrow light ranges in which they are normally used. In these ranges, they act like ideal devices with a quantum efficiency about the same as that of the eye. As the scene luminance is lowered, however, various sources of fixed noise (invariant with scene luminance) dominate and obscure the picture. These sources of noise include the noise in a television amplifier, the shot noise in a scanning beam, and the fog in photographic film. None of these sources represent absolute limits to the performance of pickup tubes or film since designs are conceivable in which these sources of noise are absent and only the intrinsic noise in the primary photo process is present. They do, however, represent present and, it is hoped, transient limitations. A further handicap to the performance of film at low illuminations is the fact that more than one absorbed quantum is needed to make a grain developable. When the incident concentration of quanta falls below the concentration of grains, the picture disappears as if by a "clipping" action. In brief, photographic film would satisfy ideal performance even, or especially, at arbitrarily low scene luminances if (a) fog were absent and (b) a single absorbed quantum were sufficient to make a grain developable. Film could then *count* each absorbed quantum.

TABLE I.

Film	Signal-to-noise ratio of 40-micron-diameter disk on film. (From Jones and Higgins.[9]) Density of film ≑0.4	Signal-to-noise ratio of image of 40 micron diameter disk at retina. Computed for 0.5 percent quantum efficiency and 0.2 seconds storage time.
Tri-X	11	13
Super XX	23	22
Pan X	36	39
Fine grain	77	58

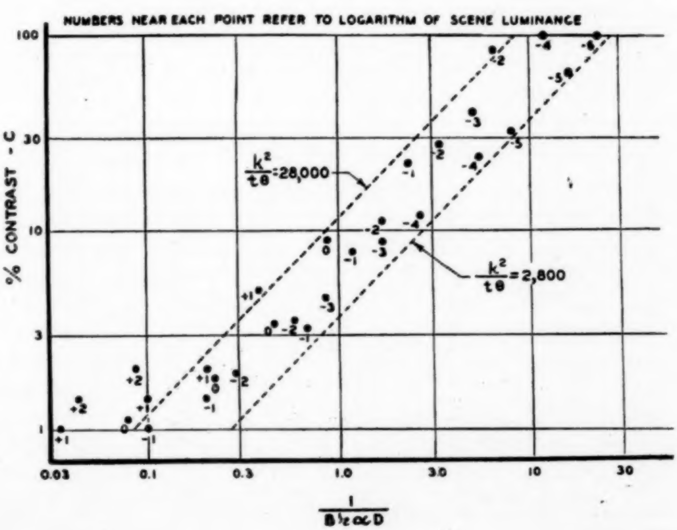

FIG. 10. A reduced plot of the data in Fig. 9. The two dotted lines are the same as the two solid lines in Fig. 8.

B. Dark Adaptation and Related Phenomena

The outstanding feature of dark adaptation is well known. Immediately after exposure to a luminance of about 100 footlamberts, the lowest luminance the eye can detect is over 1000 times larger than the luminance it can detect after extended dark adaptation. The significant question here, that bears on the mechanism of the eye, is, "Is the sensitivity, that is, quantum efficiency, of the dark adapted eye over a thousand times greater than that of the light adapted eye?**** The answer, from Figs. 8 and 10, is definitely in the negative and with a large factor of safety. From these figures, at most a factor of ten can be ascribed to change in quantum efficiency. The rest, except for some contribution of pupil opening, must come from another mechanism. And a reasonable mechanism to postulate is a gain control mechanism located between the primary photo process at the retina and the nerve fibers that carry the impulses to the brain. A gain mechanism, minus the idea of control or variability, is not at all *ad hoc*. It is needed to raise the energy level of the absorbed quanta to the energy level of their corresponding nerve pulses. To add variability to the gain mechanism is indeed a minor assumption and one that can readily account for the large range of dark adaptation.† From necessarily subjective evidence, the gain control appears to be automatically set so that noise is near the threshold of visibility. At very low lights, around 10^{-4} footlambert, "noise" appears to be more easily visible than at moderate lights around one footlambert. The writer has been most impressed by the appearance of noise in dimly lit scenes after the thorough dark accommodation that comes from several hours of sleeping in a dark room. Since these conditions are not the normal ones for making reliable observations, the reference must be regarded as one of interest but not of evidence.

At the risk of being repetitive, the conclusions of this section may be stated in another way. Photo-chemical mechanisms that are confined to the primary photo-process at the retina cannot account for more than a few percent of the total range of dark adaptation. By primary photo process is meant the process in which the incident light quanta are absorbed. The *products* of the primary photo process may however be transmitted to the nerve fibers with variable efficiency consistent with the variable gain mechanism already discussed. Thus the assumption of a variable concentration of active material whose absorption of incident light quanta is correspondingly variable, or the assumption of rods and cones with a variable threshold of excitation can be expected at most to play only a minor role in dark adaptation.

It is interesting to record here a possible but less certain application of the gain control mechanism. At high lights, luminosity, visual acuity[12] and contrast discrimination are substantially the same for red and blue illumination having the same luminance. At very low lights, less than 10^{-3} footlambert, luminosity, visual acuity[12] and contrast discrimination under red light rapidly approach zero while under blue light, significantly finite values are maintained. In the intermediate range of 10^{-3} to 1 foot lambert, the range of present interest, the luminosity of red light drops below that of blue light while acuity[12] and contrast discrimination[13] remain substantially the same for the two colors. A formal explanation of the observations in the intermediate light range follows immediately if one allows fluctuations in the primary photo process to determine visual acuity and contrast discrimination. Then, if the gain control is set high enough so that these fluctuations are apparent to the brain, all possible intelligence is thereby transmitted to the brain

**** If one takes, for example, Hecht's (reference 10) assumption that threshold visibility corresponds to a fixed amount of sensitive material decomposed by the incident threshold light, then since the threshold light intensity changes by a factor of 10^4 (see Fig. 3 of Hecht's paper, "Rod portion of the 'blue' curve"), from low to high adaptation light intensities, the quantum efficiency must also change by this factor.

[10] S. Hecht, "The instantaneous visual thresholds after light adaptation," Proc. Nat. Acad. Sci. **23**, 227 (1937).

† Parallels to the idea of a variable gain element are common in electron tubes. In the image orthicon (reference 11), for example, an electron multiplier acts as the variable gain element that raises the level of signal and noise coming out of the tube above the noise level of the amplifier to which the tube is connected.

[11] A. Rose, P. K. Weimer, and H. B. Law, "The image orthicon, a sensitive television pickup tube," Proc. I.R.E. **34**, 424 (1946).

[12] S. Shlaer, E. L. Smith and A. M. Chase, "Visual acuity and illumination in different spectral regions," J. Gen. Physiol. **25**, 553 (1942).

[13] M. Luckiesh and A. H. Taylor, "Tungsten, mercury and sodium illuminants at low brightness levels," J. Opt. Soc. Am. **28**, 237 (1938).

and variations of the gain setting vary luminosity but not acuity or discrimination. According to this argument, the gain for red light is less than that for blue light in the intermediate light range.

C. Other Mechanisms

It was stated earlier in this paper that the departure of the actual performance of the eye from that to be expected from an ideal device was a measure of the "logical space" within which one could introduce mechanisms, other than fluctuations in the primary photo process, to determine the performance of the eye. Such other mechanisms would, of course, lead to lower performance than would fluctuations in the primary photo process alone. What is important, then, is to get an estimate of the extent of this "logical space."

To clarify the problem, reference is made to Figs. 8 or 10. If independent measurements of k, t, and θ verify that $k^2/t\theta$ is 2,800 at low lights and 28,000 at high lights as shown in these figures, then, except for minor departures, the actual performance of the eye matches the performance expected from an ideal device and the "logical space" is substantially absent. The inquiry then leads to what is known of k, t, and θ separately.

The threshold signal-to-noise ratio, k, was taken from Fig. 6. Its value, 5, is primarily a low light value in that it applies to the condition that noise is easily visible. If noise is not easily visible, as at higher lights, an increase in k can be invoked. But such an increase is in the direction already noted in Figs. 8 and 10 and would only relieve the quantum efficiency (θ) of the necessity of varying from low to high lights.

The storage time (t) was also observed from Fig. 6 and the original kinescope pictures to be about 0.25 second. This value applies to the intermediate light range. At very low lights, if one takes the constant in the often quoted law of Blondel and Rey, the storage time is still about 0.2 second. Finally, the data of Cobb and Moss in the range of 1 to 100 footlamberts was taken for an exposure time of 0.18 second and match the data of Connor and Ganoung fairly well, the latter having been taken for an observation time of one second. All of this points to a storage time of 0.2 second independent of scene luminance. Langmuir and Westendorp[14] confirm this constancy except for a suggestion of a longer storage time near absolute threshold.

In spite of all of these independent sources of evidence pointing to a storage time of 0.2 second, there is still some uncertainty. The uncertainty comes from not having good data on how well the memory process can extend the physical storage time to times longer than 0.2 second. Such extension would of course vary with the observer and improve with training. Some remarks and data in Blackwell's paper suggest that memory may extend the effective storage time up to seconds. The most that may be said for the data quoted in the present paper, with the exception of the Cobb and Moss data, is that the effective storage time may be anywhere between the physical storage time of 0.2 second and the actual observation time of one second.

Independent measurements of quantum efficiency at low lights bracket the value of 5 percent used in this paper. Hecht,[15] by a statistical analysis of threshold measurements, consistently arrives at about 5 percent. Brumberg, Vavilov and Sverdlov,[16] by a similar experiment, arrive at values from about 5 to 25 percent. Both Hecht and Brumberg's measurements are for blue light in the neighborhood of maximum visual response. They should be divided by a factor of about three to reduce them to white light for comparison with the value of 5 percent already noted in this paper. At high lights, the writer knows of no independent measurements of quantum efficiency.

To summarize the discussion thus far, independent measurements of k, t, and θ agree well with the low light value of $k^2/t\theta$ in Figs. 8 and 10. At high lights there is uncertainty both about k and θ. If k increases or θ decreases, the high light value of $k^2/t\theta$ in Figs. 8 and 10 might be independently verified. In that event little room is left for mechanisms other than fluctuations in

[14] I. Langmuir and W. F. Westendorp, "A study of light signals in aviation and navigation," Physics 1, 273 (1931).

[15] S. Hecht, "The quantum relations of vision," J. Opt. Soc. Am. 32, 42 (1942).

[16] E. M. Brumberg, S. I. Vavilov and Z. M. Sverdlov, "Visual measurements of quantum fluctuations," J. Phys. U.S.S.R. 7, 1 (1943).

the primary photo process to determine the acuity and contrast discrimination of the eye. If, however, k and θ are independent of scene luminance, as much as a factor of ten in performance can be ascribed to the limitations imposed by other mechanisms.

There remains the departures from straight lines noted in Fig. 9. Since, at a fixed scene luminance, k, θ, and t should remain constant these could not account for such departures. It is rather more likely that the departures represent optical defects in the sense that, as the scene luminance is lowered, the eye combines signals from neighboring rods and cones to form larger picture elements. These larger picture elements, if they are of the same order as the smallest resolvable black disks, would limit acuity in the same way that the separate cones set a final limit to acuity. That the eye combines signals from neighboring rods and cones is a consequence of the fact that more than one absorbed quantum is needed to generate a visual sensation (see also Hecht[15]). Objective measurements by Hartline[17] on the frog's eye also point to such a combining process.

[17] H. K. Hartline, "Nerve messages in the fibers of the visual pathway," J. Opt. Soc. Am. **30**, 239 (1940).

SUMMARY

The performance of the eye over the bulk of its operating range may be matched by an ideal picture pickup device having a storage time of 0.2 second and a quantum efficiency of 5 percent at low lights decreasing to 0.5 percent at high lights. For many engineering problems in which the performance of the eye must be quantitatively compared with the performance of man-made pickup systems, the substitution of an equivalent ideal device for the eye considerably simplifies the analysis. The match between the eye and an ideal device also provides at minimum a good first approximation to an understanding of the performance of the eye in terms of fluctuations in the primary photo process. Depending mostly on how well further independent measurements of the quantum efficiency of the eye agree with the quantum efficiencies deduced in this paper, the analysis of performance in terms of fluctuations may be appreciably better than a first approximation.

ACKNOWLEDGMENTS

The writer has profited from many discussions of the subject of this paper with Dr. D. O. North of these laboratories.

A Cinema-Spectrograph for Photographing Rapid Spectral Sequences*,**

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(Received September 16, 1947)

A simple spectrograph designed around a 16-mm movie camera is described. The spectrograph was built for the purpose of obtaining spectrograms from light sources showing transient phenomena, in particular such light sources as the new types of rocket and jet combustion engines. The study of the light emitted from an acetylene-oxygen flame as a function of time illustrates its usefulness.

INTRODUCTION

WITH the advent of new types of combustion engines such as jets and rockets of various types, it has become increasingly important to study combustion reactions. Spectroscopic techniques have an important role in these studies particularly in the measurement of temperature and the identification of the diatomic free radicals. Since the nature of many of the combustion processes is such that variations occur as a function of time, spectrographic studies have been made utilizing the moving plate technique, with the plate motion parallel to the slit, so that spectral changes may be recorded during the process.[1] For example, variations in the air-fuel ratio will alter the emitted spectrum since the concentrations of the various molecular fragments will depend upon this ratio. Furthermore, in high velocity flames there are shock and turbulence phenomena which frequently cause variations in brightness in different parts of the flame at different times. Many of the flames one might wish to study are not accessible to the usual spectroscopic laboratory. In particular, experiments with rocket flames and jet motors must of necessity be conducted in large open areas or in specially constructed cubicles. Some of these flames exhibit transient phenomena over a period of time too long to be recorded on a moving plate.

It was felt worthwhile, therefore, to develop a portable spectrograph of high aperture which could be carried about to investigate various flames *in situ*. It seemed natural to design an instrument around an ordinary motion picture camera of the magazine-loading type. Such an optical set-up can at once meet the requirements of portability, ruggedness, high aperture, automatic film transport and ease of loading under strong external illumination. Certain disadvantages were, of course, immediately apparent. The linear dispersion is low, although this can be corrected to some degree by the use of telephoto optics, and the spectral range is limited to the region from ~3800A to ~6500A. The lower limit is set by the glass system and the upper limit by the photographic emulsions available for ordinary movie camera use.

CONSTRUCTION AND DETAILS

In order to make the light gathering power of the spectrograph as great as possible, it is advisable to select a dispersing system of optimum efficiency. For this purpose, prisms are, in general, to be preferred to gratings, particularly since one is restricted to refractive rather than reflective systems because of considerations of high aperture and short focal lengths. The choice, therefore, is one of prism design and a direct vision prism was decided upon, rather than a single prism, since it permitted keeping all

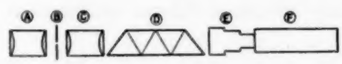

FIG. 1. Schematic diagram of plan view of the moving picture spectrograph. The condenser system, slit and collimating system are shown at A, B and C respectively. The 5-element direct vision prism is shown at D, while the moving picture camera and telephoto lens are represented at F and E.

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** A preliminary account of a portion of this paper was presented at the Symposium on Molecular Structure and Spectroscopy sponsored by the Graduate School and Department of Physics and Astronomy of The Ohio State University, June 9–14, 1947 and at the 32nd meeting of the Optical Society of America, October 23–25, 1947.

[1] For example, see B. L. Crawford and C. Huggett, Rev. Sci. Inst. 17, 213 (1946).

parts of the optical train in line. This makes for a compact instrument of small over-all dimensions. The particular prism used, an excellent 5-element direct vision prism consisting of three crown and two flint elements, was loaned to us for the purpose by Dr. I. C. Gardner of the National Bureau of Standards.[2]

The instrument first set up is shown in Fig. 1. The various elements are as follows: *A* is the condensing lens of aperture f:1.5, taken from an old delineascope; *B* is the entrance slit of fixed type (a selection of several slit widths is available); *C* is the collimating lens of aperture f:1.5, also removed from a delineascope; *D* is the 5-element direct vision prism, *E* a telephoto camera lens, and *F* is a 16-mm magazine-loading movie camera. The camera can be fitted with lenses of various focal lengths to provide a choice of linear dispersion on the film. The spring wind was removed from the camera and replaced with a synchronous motor drive designed to give a speed of 20 frames per second. A small neon lamp, operated from a 60-cycle source provides an optional timing trace. All the parts except the camera and motor drive are mounted on wood blocks which slide between a pair of wood rails. A bit of soap as lubricant permits sufficient smoothness of motion to make all the necessary adjustments. The camera and synchronous motor are mounted on a heavy steel plate and may be moved so as to set the camera in position for maximum intensity in the spectral region desired. A camera boresight is used for focusing the spectrograph. With the camera diaphragm wide open and set for infinity, the collimating lens is set so that the spectrum of a line source is in sharpest focus on the ground glass screen of the boresight. For distant sources which cannot be readily focused on the entrance slit, one can mount a slit of suitable width near the source. In this case, the condenser lens, entrance slit and collimator are removed and the camera lens is focused on the distant slit to bring the spectrum into sharp focus on the boresight screen.

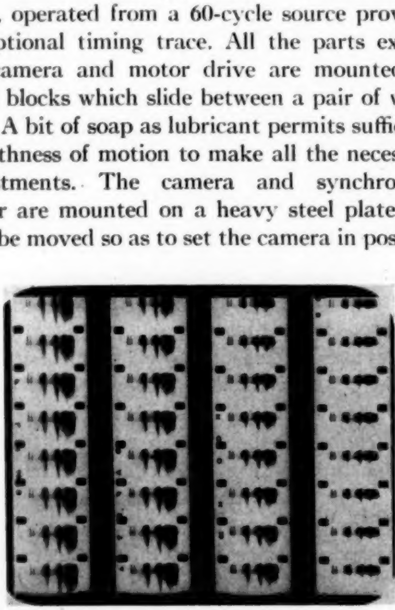

FIG. 2. Spectrograms of an acetylene flame showing the effect of varying the oxygen-fuel ratio. The sequences are from top to bottom and right to left, with the long wavelength end of the spectrum to the right.

RESULTS

The performance of the spectrograph is shown in Figs. 2 and 3. Fig. 2 consists of a sequence of movie frames, running from top to bottom and right to left, of an oxygen-acetylene flame taken as the oxygen-fuel ratio is varied. The camera-focusing lens employed was a 2″ focal-length lens of aperture f:1.5. The long narrow flame was focused along the length of the entrance slit so that various portions of the slit image correspond to the various regions of the flame. The spectrograms early in the sequence show the spectrum of the brilliant blue inner cone. The emission from the outer cone is small since the oxygen-fuel ratio is high. Several sequences of the Swan system of the C_2 molecular fragment are to be noted. These are the sequences[3] headed at 5635A ($v'-v''=-1$), 5165A ($v'-v''=0$) and 4737A ($v'-v''=+1$). In addition, there are a number of bands of longer wave length which belong to the sequence $v'-v''=-2$, and there is also the sequence $v'-v''=+2$ at 4382A. The remaining band is the (0, 0) band of the CH system at 4315A. As the flame becomes relatively richer in acetylene, the character of the flame changes and the outer cone becomes quite luminous. Microphotometer curves have been taken of several of the spectra in Fig. 2, with a Leeds and Northrup Knorr-Albers Recording Microphotometer. Typical microphotometer curves of the spectra of the inner and outer cones of the oxygen-acetylene flame are shown in Fig. 3. In

[2] The angular separation between the emergent *C* and *F* lines is 6°50′. The clear aperture of the 5-element prism is 20×20 mm.

[3] The notation employed is that given by W. Jevons in *Report on Band-Spectra of Diatomic Molecules* (The Cambridge University Press, Teddington, England, 1932) in which the single primes denote the upper state.

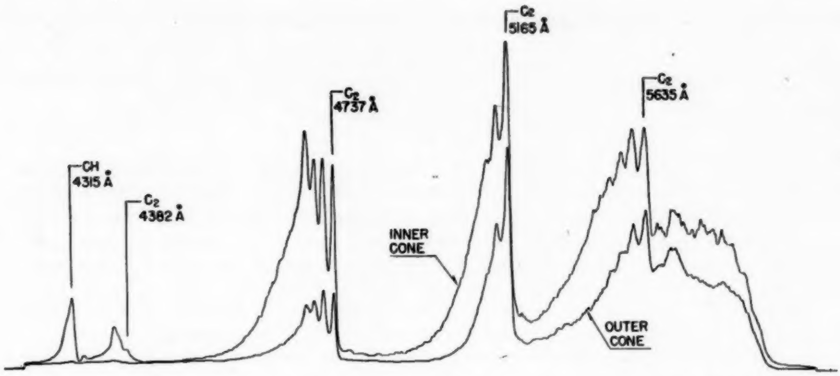

FIG. 3. Microphotometer curves of typical spectra of an oxygen-acetylene flame photographed with a cinema-spectrograph, during burning with a high acetylene-to-oxygen ratio.

this case the flame was operated with a high acetylene-to-oxygen ratio. The differences in relative intensity among the vibration bands in some of the sequences are caused by the different conditions existing in the two parts of the flame. The variations of intensity of the CH and C_2 bands might be studied best in the region of 4300A with the aid of a long-focus telephoto lens. The linear dispersion obtained in the spectra of Fig. 2, taken with a lens of 2″ focal length, are 118, 138, 215 and 317A/mm at 4350, 4500, 5000 and 5500A respectively. With regard to resolving power, several bands are resolved in each sequence on the original film as can be seen in the microphotometer curves in Fig. 3. The resolving power with a 100-micron slit is at least 100 and 400 at about 5500 and 4300A, respectively.

COMMENTS AND CONCLUSION

The effective aperture of the instrument described was limited by the prism. In fact, the aperture of the lens optics of f:1.5 was reduced to a useful aperture of about f:3.5. The film employed was Eastman Super X. This indicates that considerable improvement in speed could be achieved with a probable limit of about 200 frames/sec. for sources of brightness comparable to an acetylene flame. A 3-element direct vision prism would probably be adequate, for although its angular dispersion is only slightly greater than half that of the 5-element one, it has considerably lower light losses and it can be used with a longer focal-length camera lens. In addition, 35-mm film would also offer some advantages, but for a system of resolving power of about 100 to 400, the present linear dispersion of about 200A/mm obtained with a 2″ lens seems sufficient and allows the entire useful spectrum to be photographed on 16-mm film. A worthwhile addition would be the use of sound-track film to record simultaneously the spectrum and any accompanying sonic effects from rocket and jet motors. Some of these improvements will be incorporated in a movie camera spectrograph now being constructed.

As a final remark, it might be worthwhile to mention that color film as well as black and white film has been employed with fairly striking success on various light sources such as hydrocarbon flames, Geisler discharge tubes, tungsten filaments whose brightness was varied, absorption spectra of chemical reactions involving color changes, absorption of filters such as Didymium, as well as others. The results are aesthetic and should be useful for teaching and demonstration purposes.

ACKNOWLEDGMENTS

The authors wish to express their thanks to Dr. I. C. Gardner of the National Bureau of Standards for the loan of the prism used in these experiments and to Dr. Archie Mahan of the Naval Ordnance Laboratory for several profitable discussions. They also wish to thank Mrs. Helen S. Hopfield for her cooperation in obtaining the microphotometer curves presented in this paper.

The Polarization of Infra-Red Radiation

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(Received October 8, 1947)

A transmission polarizer for the near infra-red is described, which is small and compact and can be mounted on a spectrometer without alteration of the existing optical system. It employs a pile of self-supporting selenium films, each approximately 4 microns thick. The advantages which such a polarizer possesses over other forms are pointed out, and the factors affecting the design of the pile are briefly discussed. The method of preparing the Se films is described in detail. The percentage polarization, measured by crossing two similar piles, is better than 94 percent with 5 films, and better than 98 percent with 6 films in each pile, over the spectral region 2–14 μ; the maximum transmission is 47 percent of the incident radiation.

No absorption bands of selenium could be detected between 1 and 14 μ using a much thicker layer (52 microns).

ALTHOUGH it has been recognized for many years that substances with a high degree of internal orientation exhibit the phenomenon of dichroism in the infra-red region, relatively little use has hitherto been made of this effect as an aid to the interpretation of infra-red spectra. It is probable that this is partly attributable to experimental difficulties, since the older methods which have been suggested for the production of polarized infra-red radiation are either inefficient or inconvenient to employ. Thus the use of wire gratings as suggested by du Bois and Rubens[1] is only suitable at very long wave-lengths, while the Nicol prism which is both efficient and convenient is confined to regions below 2μ because of the absorption of calcite.

The radiation emitted from a glowing platinum strip at angles greater than 88° to the normal (Laue and Martens,[2] and Czerny[3]) is almost completely polarized, but the intensity of the radiation is low.

A high degree of polarization is possible by using radiation reflected at a suitable angle from a selenium surface, as first suggested by Pfund.[4] This form of polarizer is effective over a wide range of wave-lengths but has the following disadvantages:*

(a) The intensity is only 27 percent of that of the original unpolarized beam, even when using selenium.

(b) If the selenium is supported on a base of different refractive index, radiation transmitted by the selenium (see below) will be reflected at the surface of the base plate. As it will not fall at the correct polarizing angle, some of the unwanted component will be reflected and the beam will be to some extent depolarized.

(c) The radiation is deviated from its original direction. It can be brought back by means of two further reflections, but it is impossible to insert the arrangement in a converging or diverging beam without refocusing, which is inconvenient. In some enclosed spectrometers it is almost impossible to find room for the mirrors unless they are put outside the housing, when much of the advantage of an enclosed instrument is lost.

These disadvantages can be overcome by using the beam transmitted through a pile of selenium films.

CONSIDERATIONS AFFECTING THE DESIGN OF A PILE OF PLATES

The intensity of radiation reflected from a dielectric is given by

$$(I_\sigma)_r = I_\sigma\left[\frac{\sin^2(i-r)}{\sin^2(i+r)}\right] \tag{1}$$

for radiation with the electric vector vibrating perpendicular to the plane of incidence, and by

$$(I_\pi)_r = I_\pi\left[\frac{\tan^2(i-r)}{\tan^2(i+r)}\right] \tag{2}$$

[1] H. du Bois and H. Rubens, Ann. d. Physik **35**, 243 (1911).

[2] M. Laue and F. F. Martens, Verh. d. D. Phys. Ges. **5**, 522 (1907).

[3] M. Czerny, Zeits. f. Physik **26**, 182 (1924).

[4] A. H. Pfund, Johns Hopkins Univers. Circ. No. 4, 13 (1906).

* While this paper was being prepared, an improved form of infra-red polarizer was described by Pfund (reference 5). This reflection polarizer, though more convenient than earlier forms as regards dimensions, suffers from an even greater loss of intensity than the single reflector, and transmits only 18 percent of the original unpolarized beam, at the polarizing angle.

[5] A. H. Pfund, J. Opt. Soc. Am. **37**, 558 (1947).

for radiation vibrating at right angles to the first beam. Here I_σ and I_π are the intensities of the two polarized beams before reflection, i is the angle of incidence, and r the angle of refraction in the medium.

Figure 1 gives the relation between the reflected intensities and the angle of incidence for a single selenium surface, taking n as 2.54, this being the value corresponding to a polarizing angle in the infra-red of 68½° (Czerny[6]). At the polarizing angle there is no reflection of the π component; hence, if a pile of plates is used, the whole of this component of the originally unpolarized beam is transmitted. It is then simply a matter of providing sufficient reflections to reflect away the σ component to the necessary extent.

The correct expression for the percentage polarization produced by a pile of m plates (using the transmitted beam) is (Provostage and Desains[7])

$$\frac{I_\pi - I_\sigma}{I_\pi + I_\sigma} = \frac{m}{m + \left(\frac{2n}{1-n^2}\right)^2}. \qquad (3)$$

This expression takes account of multiple reflections, which diminish the degree of polarization, but is only applicable to cases where no account need be taken of interference between the various reflected beams. Such a case would arise if the plates were thick or irregular. An interesting example of the effect of interference on the degree of polarization obtained is described below.

From (3) it is evidently advantageous to use a material of the highest possible refractive index, to obtain the highest polarization for a given number of plates.

METHODS OF MAKING PILES OF PLATES

Two methods have been employed. The first has been discarded in favor of the second method, but it is described because it is of some interest:

(1) A pile of plates can be made by depositing on a sodium chloride plate alternate layers of, for example, sodium fluoride ($n \simeq 1.3$) and thallium iodide ($n \simeq 2.5$).

The radiation beam must fall on the thallium iodide-sodium fluoride interface at the polarizing angle, and if it then falls on a parallel face separating the sodium fluoride from air, it will be totally internally reflected. In order, therefore, that the beam can emerge from the arrangement (and also to allow it to enter), prisms of suitable angle are cemented in optical contact with the layers, or the plate on which they are carried, as in Fig. 2a.

The interesting feature of this method is that, for a given wave-length, the thickness of the layers can be chosen so that all the reflected waves reinforce each other, which has the effect of increasing the reflectivity for the polarized component of the radiation which it is desired to remove. This increases the efficiency very greatly. A prism of this kind was constructed for the visible region, with layer thickness chosen to give maximum reflection in the green. It was found that with only three layers, a very good polarizer was obtained. The polarization was imperfect at the blue end of the spectrum, as the thickness was unsuitable there.

It was found difficult to apply this method to the infra-red region, chiefly because of the difficulty of making good non-scattering layers of the necessary thickness. It would also have been

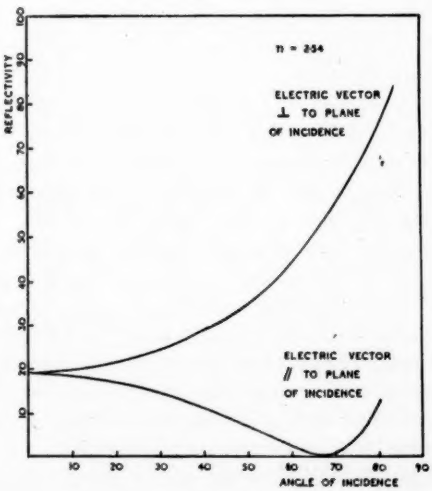

Fig. 1.

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[7] Provostage and Desains, Ann. Chim. Phys. **30**, 159 (1850).

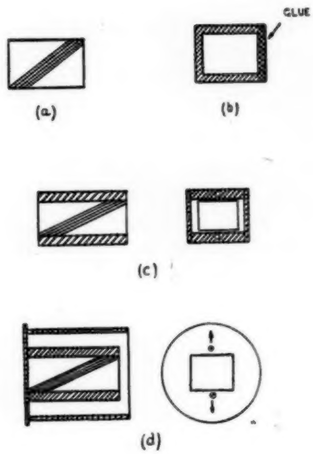

FIG. 2.

necessary to have several prisms in order to cover the required range of wave-lengths.

During these experiments selenium was used in place of thallium iodide, and it was noted that good films of considerable thickness could be made. It was also noticed that a thick selenium film on a base of sodium fluoride readily became detached from the base, suggesting that an unsupported film of selenium could be made. This led to the second method of making a pile of plates for polarizing infra-red radiation.

(2) Amorphous (precipitated) selenium is used, and in our experiments material supplied by the British Drug Houses is found to be suitable. It is contained in a small Pyrex crucible, electrically heated by a coil of Nichrome wire, and is melted in air before the evaporation is carried out in a high vacuum. Equipment of the type usually employed for depositing aluminum etc., in vacuum is suitable. The selenium is condensed on a thin cellulose nitrate film stretched over a metal ring six inches in diameter. It is necessary to protect the uncoated surface of the film from stray selenium vapor by a metal baffle plate attached to one side of the ring. The cellulose nitrate film is made from a solution of nitrate film base (having removed the gelatine coating by boiling in water) in amyl acetate, at a concentration of 3 percent.

Two 28 S.W.G. wires are stretched across a glass plate, about ten inches apart, and in contact with the glass. A pool of solution is poured on the glass, and this is swept into a uniform coating by drawing a steel straightedge across the plate, the edge resting on the wires. The dried film is easily removed from the glass after applying a jet of wet steam. It is at once applied to the ring and cemented with the cellulose nitrate solution. Subsequent contraction causes the film to become tight and flat.

Evaporation is carried out slowly, until the film of selenium shows a slightly greater density than a Wratten 29 filter. After cutting the film from the metal support, it should be stuck down (at the edges only) to a piece of paper, selenium-coated side up.

The supports for the film (which should have been prepared ready for the final stage) may consist of brass foil 0.005 inch thick, cut as shown in Fig. 2b. All the edges should be smoothed with 00 emery paper. A support is taken, and a cold glue such as Le Page's is applied to one face, along one side only. This support is then laid, glued side down, on the selenium film, and the other supports are similarly treated. When the glue is set, the films are cut round the metal supports with a razor blade, and turned over. The edge opposite the glued edge is then stuck down with cellulose nitrate solution, and the films are given a few minutes in which to dry.

The next process is the removal of the cellulose nitrate film. The support with attached film is held on the glued edge, and placed vertically in an empty vessel with rubber tube attached so that the vessel can be filled slowly from the bottom. Acetone is now slowly run in until the film is covered. If left for about fifteen minutes, the cellulose nitrate will be completely dissolved off, leaving the selenium film attached to the support at the upper edge only. The acetone reservoir is now lowered and the vessel emptied of acetone. At the same time the film is gently stretched by the surface tension forces over the frame, and can be removed.

The polarizer is made up of a suitable number of films (say five) on their holders mounted in a frame so that radiation falls on them at a mean angle of 65° (Fig. 2c). This avoids the extreme foreshortening which occurs if the correct angle of $68\frac{1}{2}°$ is used, and in practice does not appear to make any appreciable difference. The diminution in performance if the radiation is a conical beam of semi-angle 5° instead of a parallel beam is hardly noticeable. The polarizer is conveniently mounted so that it can be rotated (Fig. 2d).

The films are very fragile. It is desirable to have separators between the film holders to avoid contact between the films.

PERFORMANCE OF SELENIUM POLARIZER

The chief advantages of the polarizer made from selenium films may be summarized as follows:

(a) The arrangement is small and compact.

(b) Because of the small thickness of the films (4 microns) there is no distortion of the radiation passing through them, even though they are not flat. No refocusing is needed when the polarizer is inserted.

(c) The plane of polarization can be rotated by rotating the pile, without change in direction of the transmitted beam.

(d) The efficiency is high (see below). The fact that the films are not flat improves the efficiency, as the effect of multiple reflections is lessened.

PERCENTAGE POLARIZATION

The percentage polarization produced by a pile of five films each of 4-microns thickness has been measured by crossing this pile with a similar one in front of an infra-red spectrometer. If the intensity of the radiation vibrating in and perpendicular to the plane of incidence of the polarizer be I_π and I_σ, respectively, the percentage polarization is defined as $(I_\pi - I_\sigma)/(I_\pi + I_\sigma)$. In the region 2–14$\mu$, the percentage polarization produced by five films is never less than 94 percent, and over most of the region it is much nearer 100 percent. There is a small fluctuation in percentage polarization with wavelength. This is caused by the greater reflection of the unwanted vibration at wave-lengths at which the reflections from the two surfaces of a selenium film reinforce each other.

If the number of films is increased to six, the percentage polarization is better than 98 percent over the range 2–14μ. Doubtless the polarization extends to much greater wave- lengths.

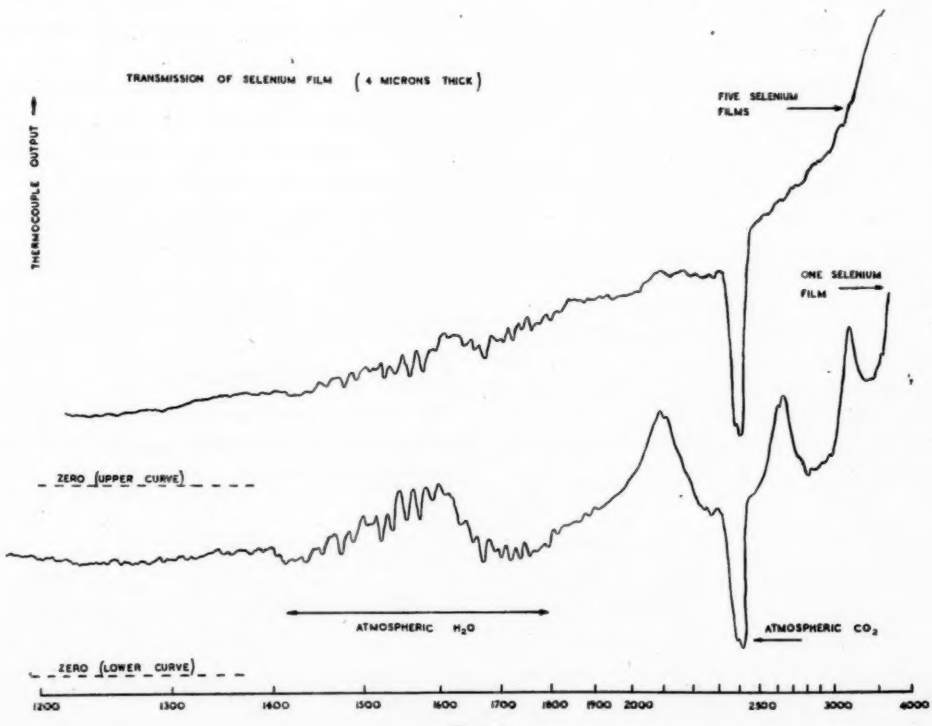

FIG. 3.

It is evident that the more perfect the polarization of the transmitted beam, the smaller are the effects of interference on transmission. A completely polarized beam, having zero reflection at the polarizing angle, cannot give rise to the separated beams necessary to produce interference. As may be seen from Fig. 1, the reflection is small at 5° on either side of the polarizing angle.

The disappearance of interference effects with increasing perfection of polarization is shown by Fig. 3, which gives the spectral energy curve of part of the infra-red spectrum of a Nernst filament first, with only one selenium film interposed at the polarizing angle, then with the pile of five films. This record has been made with a spectrometer in which the slit is automatically opened as the wave-length increases, to compensate for the fall off in energy radiated by the Nernst filament.

TRANSMISSION OF PILE

When the film is large enough to transmit all the radiation cone which the spectrometer can accept, the transmission of the polarizer (five films) is 47 percent of the original unpolarized beam, i.e., 94 percent of the required component is transmitted. However, because of the polarizing effect of the spectrometer prism and mirrors, the radiation transmitted to the thermocouple is cut down unless the most favorable orientation of the polarized beam is used.

In order to have equal energy transmission when the polarizer is rotated through 90°, it is necessary to set the vibration plane of the transmitted beam at 45° to the spectrometer slit. With the particular spectrometer employed, the transmission was then 40 percent of the original unpolarized beam (with five films). If a polarizer of six films is employed, the transmission is 37 percent.

TRANSMISSION OF SELENIUM

It is evident from what has been said that selenium in thin films is very transparent in the infra-red. An examination of the transmission over a wide range of wave-lengths has been made, with a considerably increased film thickness (52 microns).

In order to avoid complication arising from interference fringes, selenium was deposited on a plate of a "mixed crystal" of thallium bromide and iodide ($n=2.38$ approx.). This material matches the refractive index of selenium sufficiently well to make the plate and film practically optically homogeneous. Since the plate was 2 mm thick, no effect of interference fringes was observed. Over the range of wave-lengths 1–14μ, the absorption of a layer of amorphous selenium 52 microns thick was too small for measurement.

It may be noted that polarized radiation could be used to eliminate interference fringes, produced when the absorption spectra of thin uniform films are examined. If the radiation (polarized in the appropriate direction) falls on the film at the polarizing angle, no reflection, and hence no interference, will occur. This method has the disadvantage that in order to calculate the path traversed in the film, the refractive index for infra-red radiation must be known. The method might, however, be useful when the absolute values of the absorption coefficients are not required, but where the simplification resulting from avoiding interference fringes is useful.

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